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MECHANISM OF SYNTHESIS OF SUPERHEAVY NUCLEI VIA THE PROCESS OF CONTROLLED ELECTRON-NUCLEAR COLLAPSE

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This paper presents a brief review of the existing approaches to the creation of superheavy nuclei in collisions of heavy nuclei to overcome the Coulomb barrier or through the pion condensation in a nucleus volume.

A principally new approach to the creation of superheavy nuclei based on the stimulation of a self-organizing collapse of electron-nuclear systems is analyzed. For a neutral atom compressed by external forces, a threshold electron density is shown to exist. If such a density is reached, a self-organizing process of "electron downfall to the nucleus" starts. This process is exoenergic and leads to the formation of a supercompressed electron-nuclear cluster. The higher the charge of a nucleus, the lower the threshold of the external compression. It is shown that the maximum binding energy shifts during such a self-organizing collapse of the electron-nuclear system from $A_{\rm opt} \approx 60$ (for uncompressed substance) to the area of high mass numbers $A_{\rm opt} \geq 200 \ldots 2000$ and could render the synthesis of superheavy nuclei to be energy-efficient. The synthesis proceeds through the absorption of other nuclei by the collapsed nucleus. It is theoretically proved that the synthesis efficiency is ensured by both the width reduction and increased transparency of the Coulomb barrier in the extremely compressed electron-nuclear system. The release of binding energy through the absorption of nuclei by the electron-nuclear collapsed clusters may result in the simultaneous emission of lighter nuclei. It is assumed that just such a mechanism of synthesis explains the creation

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of superheavy and other anomalous nuclei observed in the experiments carried out at the Electrodynamics Laboratory "Proton-21."

Key words: superheavy nuclei synthesis, electron-nuclear collapse, electron-nuclear cluster, degenerate electron gas, Coulomb interaction.

1. INTRODUCTION

One of the most important parts of modern physics is the study of anomalous states of substance. The creation of anomalous and superheavy nuclei is of particular interest for science and practical applications as well.

The creation of superheavy nuclei by the collision of accelerated heavy nuclei is carried out in several laboratories and requires the unique heavy-particle accelerators and the huge financial and material expenses. Inefficiency of this technique and the fatal problems with the interpretation of results (e.g., Refs. 1, 2) are the consequences of the method's inherent drawbacks.

The basic concept of the method of creation of superheavy nuclei via collisions of heavy particles has a direct "force" nature and requires to collide two heavy high-energy nuclei by overcoming the Coulomb barrier, which creates a compound nucleus with the mass close to that of colliding nuclei. It is easy to prove that the creation of superheavy nuclei, based on this concept, is extremely inefficient and limited.

Below, we present some relevant reasoning:

1. A very small probability of the frontal impact of a nucleus accelerated up to the high energy to overcome the Coulomb barrier results in a very small probability of the formation of a compound nucleus. If we compare the synthesis cross-section of a superheavy nucleus ($\sigma_f \approx 10^{-10}$ barn) and the ionization cross-section of target atoms ($\sigma_i \approx 10^8$ barn), it becomes clear that the overwhelming part of the kinetic energy of such a nucleus is spent for the ionization of target atoms.
2. A heavy compound nucleus synthesized from two lighter nuclei with the proton-neutron ratio close to normal is deficient in neutrons and has a great surplus of protons. A too low neutron number renders the nucleus to be extremely unstable and unobservable because of its short half-life.
3. The high kinetic energy of relative movement which is necessary to overcome the Coulomb barrier plays a negative role for the formed compound nucleus. This excessive energy results in the overheating of nucleons of the compound nucleus, fast emission of neutrons, protons, and clusters, and nucleus decay.

As a result, the extremely small amount of superheavy nuclei is formed and detected. The typical detection rate of nuclei for $Z \approx$

$112\ldots118$ by their decay products does not exceed 1 nucleus per month (the number of background events in the system is about $10^{11}\ldots10^{12}$). Naturally, such a detection efficiency makes any detailed study of such nuclei and, furthermore, their use impossible. In particular, the half-life measurement error for such superheavy nuclei is tremendous.

Moreover, these factors become more and more essential with increase in the charge and mass of synthesized superheavy nuclei.

It is clear that these circumstances impose heavy restrictions on the synthesis of superheavy nuclei and make the formation of even a single nucleus with $A > 300$ by the collision-based technique extremely hard. It is obvious that such an approach is not suitable for the creation of a macroquantity of superheavy nuclei and their practical use, in particular.

It should be noted that the mentioned sources of instability of synthesized superheavy nuclei are inherent to the method of their creation only and have no relation to the problem of stability of superheavy nuclei with optimized structure.

Is is worth noting that the synthesis of superheavy nuclei with $Z > 100$ by a reactor-based technique (the neutron bombardment of stable heavy nuclei) is also extremely inefficient because of the alternate ways of transmutation and the fast decay of the activated nucleus during a time interval between consecutive neutron captures.

The alternative approach to this problem was discussed in a number of works (Refs. 3–5) devoted to the creation of superheavy nuclei by pion condensation in the nucleus volume.

In the 1970s–1980s, A. Migdal and his colleagues considered the prerequisites for the creation of quasistable superheavy nuclei with charge and mass number in the range from $Z \geq (\hbar c/e^2)^{3/2} \approx 1600$ and $A \geq 3000$ to $A \geq 200\,000$, and the creation of anomalous superdense nuclei with $A \approx Z/2 \geq 100$ (see Refs. 3–5). As a basic mechanism of nucleus stability, Migdal considered the influence of a pion condensate in the volume of such nuclei.

In brief, the idea of the pion influence on the stability of a nucleus looks as follows. The pions stabilizing a superheavy nucleus may be formed in the reaction of spontaneous decay $n \rightarrow p + \pi^-$. This reaction may take place only in a very deep potential well formed by protons in two types of nucleus: in a superdense nucleus or a nucleus with high charge $Z \geq (\hbar c/e^2)^{3/2}$.

The accumulation of negative pions results in their condensation at the lowest energy level. The influence of pions on the stability of a nucleus involves two consistent mechanisms.

First, the negative pions screen a part of the Coulomb potential created by protons. Since the total number of protons exceeds the number of pions, such a mechanism, however, may not provide the full screening and final stability of a nucleus. The final stability of a nucleus in the presence of the pion condensate is provided by the specific quantum-mechanical square-law effect of attraction. This attraction does not depend on the sign of a charge and occurs when the energy of a particle (a pion) in the very strong Coulomb field of protons inside a nucleus exceeds its rest energy $m_\pi c^2$. For the average distance between protons inside a nucleus $\langle r_{pp'}\rangle \approx 1.3(A/Z)^{1/3} \times 10^{-13}$ cm, only light particles (pions) may provide such an attraction. It is obvious that the attraction of a light pion to several neighbor protons is equivalent to the mutual attraction of these protons. On the other hand, heavier protons at the same distance may not provide such a nonlinear mode of mutual attraction. Hence, each pion stabilizes a superheavy nucleus twice: as a negatively charged particle and as a lighter (in comparison to a proton) particle participating in the specific quantum-mechanical interaction.

Moreover, not only negative pions but also positive ones stabilize the nucleus.

Some additional complication of these processes which does not change the main result (the stabilization of a nucleus) arises from the collective interaction of protons with the pion Bose-condensate (see Refs. 3, 4).

It is clear from the scenario considered that the efficiency of the stabilization mechanism of nuclei by pions grows with additional compression of a nucleus (i.e., with increase in both the nucleon density and Coulomb field inside a nucleus) or with increase in the initial charge of a nucleus. In this case, the increase in the intranuclear Coulomb field and the repulsion of protons are overcompensated by the stronger nonlinear increase in the efficiency of quantum-mechanical attractions with participation of pions. It follows from the analysis in Refs. 3–5 that the energy efficiency of the self-compression of a nucleus may result in a final increase in the nucleon density in a nucleus by 3-6 times, which meets the balance of the considered attraction and repulsion forces between the nucleons at a very short distance owing to the strong interaction.

The stabilizing presence of a pion condensate allows one to predict the stability of a nucleus with pion condensate in the mass range $A_{\text{opt}} \geq 200\,000$ (see Refs. 3–5).

The general dependence of the nucleus energy on the mass number in the presence of a pion condensate (estimated per one nucleon and normalized to the iron nucleus) is presented on Fig. 1. The first potential well with a minimum at $A_{\text{opt}} \approx 60$ corresponds to the area 1 of rather "ordinary" nuclei with $A < 250$. This area is separated by a very high barrier (area 2) from the second much wider and deeper well with a minimum at $A_{\text{opt}} \approx 200\,000$ (area 3). Since the bottom of the second well is by 15-20 MeV deeper than that of the first, any nucleus "transferred" from area 1 to area 3 became unstable relative to an increase in A during the pion condensate formation and may repeatedly absorb "ordinary" nuclei, increasing its own mass up to A_{max}.

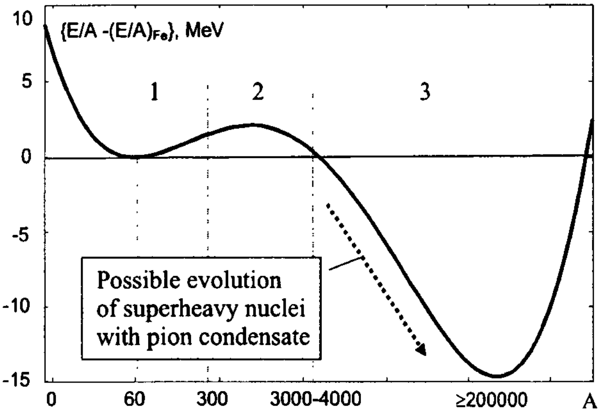

Fig. 1. Probable scheme of the nucleus energy dependence on the mass number A at the normal nuclear density without pion condensate in the nucleus (area 1) or with it (area 2). The arrow in the figure shows the possible evolution of superheavy nuclei with pion condensate.

The main problem is how to overcome the energy barrier in area 2 and achieve the conditions for the appearance of a pion condensate. Migdal considered that the height of this barrier may be so great that the spontaneous tunnelling transition may not happen during the existence of the Universe. A certain external influence is necessary. According to the Migdal's model, the transition through area 2 from the first to the second well is possible on the basis of any of two conditions: a very high charge of the initial nucleus ($Z \geq 1600$) or a pulse isotropic compression of the nucleus up to nucleon densities higher than the initial one by $3\ldots6$ times. If, in the area of the second potential well, the same relation between A and Z like that for $A < 300$ is applicable, then the mass of a nucleus with such a charge will be $A \approx 3400\ldots4000$.

Clearly, it is extremely difficult to satisfy both these conditions.

It is clear from the above analysis that neither the accelerator technique nor pion condensation can provide synthesis of superheavy nuclei with $Z > 120$ and $A > 300$.

Below, we propose a basically different method for the synthesis of superheavy nuclei with $120 < Z < 1600$, $300 < A < 3000\ldots5000$ which does not require superaccelerators and other supertechnologies.

The method is based on the simple idea of providing such initial conditions which render the process of synthesis energy-efficient and essentially increase its probability owing to the self-compressing of the electron-nuclear plasma in the process of "electron downfall to the nucleus".

Below, it is shown that this process requires a certain threshold density of the degenerate relativistic neutralized electron plasma that depends on the nucleus charge. The lowest threshold corresponds to nuclei with the largest charge. If such a density is attained by the action of external drivers, then the plasma pressure caused by the Coulomb attraction of electrons to the nucleus will be more stronger than the Fermi kinetic pressure of electrons which prevents the "electron downfall to the nucleus".

In this case, the collapse-like self-compression of the electron environment nearest to the nucleus occurs and results in the screening of the Coulomb barrier in the interaction of nuclei.

The effect of self-compression of the electron environment of a nucleus increases with the nucleus charge. During such a self-compression, the full energy of the system consisted of "a nucleus and electrons which compensate its charge" decreases essentially (and the binding energy increases, respectively).

The energy decrease is more essential with increase in the nucleus charge, which stimulates the synthesis of heavier nuclei.

In the frame of such a process, the energy of an initial driver is necessary only for getting the threshold density of the degenerate electron gas, and the further compression is going on due to the Coulomb attraction of relativistic electrons to the nucleus.

Owing to these two effects (screening and increase in the binding energy), the synthesis of nuclei with large Z and A in the volume of the self-compressed degenerate electron gas becomes efficient and has a great probability.

This collapse of the degenerate electron gas in the Coulomb field of a nucleus is analogous to the phenomena of gravitation collapse of an astrophysical object with the mass being more than the critical one, and the shell of a degenerate relativistic gas is neutralized by the charge of nuclei.

The present paper considers subsequently the proposed method of synthesis of superheavy nuclei in detail.

Since the process involves both the nucleus and the adjoining system of degenerate electrons, these systems are thoroughly analyzed.

Section 2.1 considers the general principles of the interaction between a compressed electron gas and nuclei in the whole range of pressures and substantiates the method of initiation of the synthesis of superheavy nuclei which is based on the influence of the electron subsystem.

In Sec. 2.2, the expression for the full interaction energy of electrons with the nucleus is deduced on the base of Dirac equation with regard for both relativistic and nonlinear effects.

Then the analysis of collective effects leading to the Coulomb repulsion of electrons, interchange of spin processes, and Fermi kinetic pressure of the degenerate electron gas is presented. Peculiarities of the evolution of the previously compressed relativistic degenerate electron gas in the volume of a Wigner-Seitz cell with arbitrarily charged nuclei as well as the initial stage of electron-nuclear collapse are examined.

Section 2.3 determines the conditions for the energy-efficient synthesis of superheavy nuclei which are considered in the frame of the universal hydrodynamic model of a nucleus in the presence of a degenerate electron gas. The expressions for parameters of the modified "stability line" of nuclei with minimum at $Z \gg 60$ were obtained, and the influence of electrons on the Coulomb barrier was calculated.

The main idea of the considered method consists in the proper use of degenerate electrons as a catalyst for the synthesis of superheavy nuclei. Exactly they ensure such a screening of the nucleus charge and the increase in the binding energy of the electron-nuclear system which are sufficient for the fast energy-efficient synthesis of superheavy nuclei.

The main conclusion is that any consideration of nuclear processes occurring in superheavy nuclei is impossible in principle without taking into account the influence of the electron environment.

In our opinion, this mechanism explains the synthesis of the stable superheavy and other anomalous nuclei observed in experiments (see Refs. 6–9) with supercompressed condensed substance which were carried out without nuclear reactors and superaccelerators.

2. THE MECHANISM AND THRESHOLD CONDITIONS FOR THE ELECTRON-NUCLEAR COLLAPSE OF A HEAVY NUCLEUS IN DEGENERATE ELECTRON PLASMA

2.1. Coulomb Interaction of a Nucleus with Electrons as a Basic Mechanism of Initiation of the Synthesis of Superheavy Nuclei

It is obvious that the mechanism of initiation of the synthesis should explain consistently not only the possibility of the synthesis of superheavy nuclei, but also its high rate and the stability of a synthesized nucleus. It is clear that the optimum mechanism of initiation of the synthesis of superheavy nuclei should meet the threshold conditions in the range of charges $120 < Z < 1600$ not by a force impact on a nucleus that is proportional to the expected final effect, but by the initiation of an internal self-organizing process in the substance at the nuclear and subnuclear levels. Thus, in the ideal situation, the external impact is required only during the initiation phase, and then the global transformation of nuclear matter under its own self-similar laws with the use of internal energy sources starts. Hence, the role of the first phase in the nuclear transformation is similar to the role of the first photon in a laser generation.

In our opinion, the electron subsystem plays the key role during the internal self-organizing process at the nuclear and subnuclear substance levels and leads to the highly probable synthesis of superheavy nuclei in a certain range of conditions.

This statement is based on several facts:

1. The electron system of each nucleus is adapted to this nucleus in the best possible way owing to adiabatic conditions which allow one to consider the electron-nuclear interaction ideally symmetric, coherent, and efficient.
2. According to quantum mechanics laws, electrons in the nucleus field in "the atomic form of substance" are always in the inverse state and possess a very high potential energy relative to a nucleus even at the basic level 1 s of any atom .
3. Under certain conditions, this energy may be released in the process of "electron downfall to the nucleus" and may be utilized in an internal self-organizing process in the substance for its transition from "the atomic form" to the state of collapse of the degenerate electron-nuclear plasma.

Below, we consider a "nonforce" mechanism of electron-induced initiation of self-organizing nuclear transformations occuring with high efficiency in the range $120 < Z < 1600$ with regard for the barrier which separates "ordinary" nuclei from those spontaneously formed in a pion condensate. This mechanism can efficiently function at even smaller values of Z (including $Z \leq 92$) as well.

Let's view this mechanism in detail.

The problem of evolution of any nuclear system is usually examined on the basis of models which take into account the strong interaction between all nucleons and the Coulomb interaction between protons. In nuclear physics, a direction of reaction (fusion or fission) is defined by its energy efficiency.

The nucleus energy E_n can be expressed with sufficient accuracy and generality (without any specific definition of a nucleon shell structure in the frame of the liquid drop model) with the help of the Weizsäcker formula, where each term relates to a separate kind of interaction

$$E_n = Zm_pc^2 + (A-Z)m_nc^2 - \varepsilon_1 A + \varepsilon_2 A^{2/3} + \varepsilon_3 Z^2/A^{1/3} + \varepsilon_4 (A/2 - Z)^2/A + \varepsilon_5/A^{3/4}. \tag{1}$$

Here, $\varepsilon_1 A$ — binding energy of all nucleons; $\varepsilon_2 A^{2/3}$ — surface energy of the nucleus; $\varepsilon_3 Z^2/A^{1/3}$ — Coulomb repulsion energy of protons in the nucleus; $\varepsilon_4(A/2-Z)^2/A$ — the symmetry-involved energy owing to the rise of the inequality of protons and neutrons because of the necessity to populate high-energy levels according to the Pauli principle separately for neutrons and protons; $\varepsilon_5/A^{3/4}$ — term which takes into account pairing effects for even-even, even-odd, and odd-even nuclei.

Many variations of the given dependence exist. However, in all cases, the probability of a significant effect (not just a small disturbance) of the external electron environment on nuclear processes has not been considered. As a rule, such situation is caused by the fact that the small electron density makes their binding energy with a nucleus very small on the scale of intranuclear processes.

Quantum mechanics explains the reason for such a situation.

Let's consider two extreme cases describing the different aspects of the effect of a nucleus on the movement of electrons.

Consider the system, which consists of a nucleus and electrons not subjected to the external influence. Only those quantum-mechanical states are probable in it, which meet the solution of the Schrödinger or Dirac equations involving the Coulomb field of the nucleus. It follows from these equations that the maximum binding energy of an electron with a nucleus,

$$E_{\max} = Z^2 e^4 m_e / 2\hbar^2, \tag{2}$$

corresponds to the deepest allowed state (the state 1s in atoms). For the heaviest of stable nuclei ($Z \approx 92$), the binding energy of 1s electrons does not exceed $E_{\max} \approx 200\,\text{keV}$. The binding energy of the rest of electrons is much lower (by 4 and more times). In ordinary atoms (at $Z < \hbar c/e^2 \approx 137$), a high binding energy of electrons with a nucleus is impossible. It is obvious that, in such systems, the effect of electrons on the balance of forces determining the evolution of a nucleus is insignificant. It is also clear that such a situation is a direct consequence of the initial problem statement: the system under study consists of a nucleus interacting with free electrons according to the Coulomb law (the main interaction), and any influence of the environment (e.g., external pressure upon an electron shell) is assumed to be weak and insignificant in comparison with the Coulomb interaction inside the atom.

Let's consider the basically different, alternative case. Consider a system of nuclei and electrons under a so strong external impact that it (rather than the electron-nuclei interaction) unambiguously determines a behavior of the electrons.

The transition between the considered cases of weak and extremely strong external impact corresponds to several consecutive events.

At zero or very weak external pressure, electrons and nuclei exist as bound atoms. With increase in the pressure, the external electron shells of atoms are destroyed. The external shells completely collapse at such a density of atoms, when the distance $2R$ between their nucleus becomes equal to the double radius $r_1 = \hbar^2/m_e e^2 \approx 0.5$ Åof the external electron shells. It corresponds to the ion density $n_i \approx (m_e e^2/2\hbar^2)^3 \approx 10^{24}\,\text{cm}^{-3}$. As the pressure increases further and the ion density of the compressed plasma reaches $n_i \approx Z(m_e e^2/\hbar^2)^3 \approx 10^{25} Z\,\text{cm}^{-3}$, the majority of internal atomic shells are destroyed (see Ref. 10).

At that greater pressure which corresponds to the ion density $n_i \approx 10Z(m_e e^2/\hbar^2)^3$, the full destruction of the atomic structure occurs at all levels including the deepest ones. The nonrelativistic electron–nuclear plasma with electron density $n_e \geq 10Z^2(m_e e^2/\hbar^2)^3 \approx 10^{28} \ldots 10^{30}\,\mathrm{cm}^{-3}$ corresponds to this state.

This plasma is degenerate in the electron component if its temperature is lower than the degeneration temperature $T_{deg} \approx 350\,\mathrm{keV}$, equal to the Fermi energy $E_F = (3\pi^2)^{2/3}(\hbar^2/2m_e)n_e^{2/3}$. At even a greater compression (at the density $n_e \geq 10^{31}\,\mathrm{cm}^{-3}$), the electron component of the plasma will correspond to the relativistic degenerate electron gas. Furthermore, at $n_e \gg 10^{31}\,\mathrm{cm}^{-3}$, it corresponds to the superrelativistic gas. Noteworthy, that the density $n_e \approx 10^{30} \ldots 10^{31}\,\mathrm{cm}^{-3}$ is very high on the scale of the existence of ordinary atoms ($\langle n_e \rangle \approx 10^{23} \ldots 10^{24}\,\mathrm{cm}^{-3}$ is normal). But it is very small in comparison with the nucleon density $n_n \approx 10^{38}\,\mathrm{cm}^{-3}$ in a nucleus.

Being received as a result of the external compression of some volume of a substance, the neutral electron–nuclear plasma is degenerate in the electron component and is characterized by several types of the internal interaction between its components:

1. Interaction between nucleons in a nucleus. The full energy of this interaction is determined by last five terms in Eq. 1 for E_n.
2. Nuclei interact with the nearest electron environment and attract electrons. Moreover, the interaction of nuclei establishes a long-range ordering in the electron-nuclear plasma.
3. Electrons interact according to their spin and charge.

Let's examine the last two types of interaction in detail.

At high electron densities with regard for the interaction between charged nuclei, the minimal energy of such a system corresponds to the formation of a bulk body-centered cubic lattice of nuclei in the degenerate electron plasma (e.g., Ref. 11).

If the density of electrons neutralized by nuclei satisfies the condition $n_e \gg Z^2(m_e e^2/\hbar^2)^3 \approx 10^{25} Z^2\,\mathrm{cm}^{-3}$, then electrons behave as the ideal degenerate Fermi-gas (see Ref. 12).

Let's consider the electron-nuclear plasma as a system consisting of periodically located Wigner–Seitz elementary cells with a nucleus at the center of each cell (Fig. 2). An external pressure compressing the plasma and holding it from dispersion is applied to the system boundary. The same pressure is applied to the surface of each cell which represents a compressed quasiatom where the state of electrons is formed mainly by the external pressure. The electrostatic interaction between different cells can be neglected. The radius of each cell, $R = (3Z/4\pi n_e)^{1/3}$, is determined by its electric neutrality (the number of electrons in it equals the number of protons Z in the nucleus at the center of a cell).

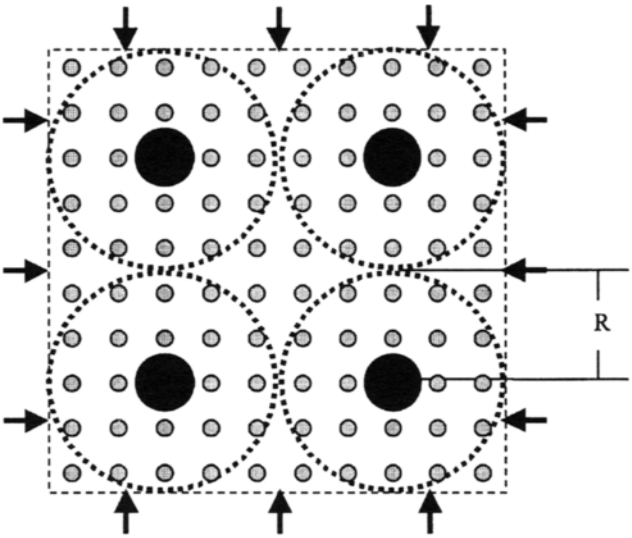

Fig. 2. System of nuclei with charge Ze in the degenerate electron gas with density n_e.

Could the interactions between all particles in the volume of a neutral Wigner–Seitz elementary cell with degenerate electron gas result in the stability of a compressed quasiatom or in its self-compression without external pressure? Such a question was examined for the first time by Salpeter in Refs. 13, 14 while solving the problem of stability of cosmological objects (black holes, white dwarfs, and neutron stars).

As shown in Refs. 13–16, the presence of the electron-nuclear attraction in a preliminarily compressed degenerate nonrelativistic electron gas results in the Fermi reduction of the electron gas pressure by several percents, which is not sufficient, obviously, for the plasma stabilization and, what is more, for self-compression. For the sake of justice, it is necessary to mention Ref. 15, where an estimation of the role of the Coulomb interaction for a compressed relativistic degenerate gas was performed with the same negative result. However, one should not consider this estimation to be valid because the authors used the nonrelativistic expression for Coulomb interaction energy $U = -e\varphi(r) = -Ze^2/r$ in the calculations based on the equation of state of the relativistic degenerate gas. This problem will be examined in more details below.

On the other hand, the detailed analysis of the interaction of a nucleus with a relativistic degenerate gas without preliminary external compression in Ref. 4 allows one to conclude that, in a nucleus with $Z \ll 1600$, the presence of the electron environment corresponds to an ordinary (not compressed) atom.

Thus, the problem of evolution of a quasiatom preliminarily compressed by external forces up to the state of a relativistic degenerate gas seems to be unexplored. The further analysis shows that this very system is capable under certain conditions to transfer to the state of self-organizing electron-nuclear collapse whose evolution facilitates the formation of superheavy nuclei.

2.2. The Energy of Interaction of a Compressed Relativistic Degenerate Gas of Electrons with a Nucleus in the Volume of a Neutral Wigner–Seitz Cell

Let's calculate the energy of interaction between all particles in the volume of a neutral Wigner–Seitz elementary cell constructed near one of nuclei.

At first, we calculate the energy of interaction of one electron with a nucleus and other electrons. In general, the movement of an electron in a field with potential $\varphi(r)$ is described by the Dirac equation

$$\{E + e\varphi(r) - \hat{\beta} m_e c^2 - \hat{\boldsymbol{\alpha}} c \mathbf{p}\}u = 0, \tag{3}$$

$$\hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{1} & 0 \\ 0 & -\hat{1} \end{pmatrix}, \quad \hat{\boldsymbol{\alpha}} = \begin{pmatrix} 0 & \hat{\boldsymbol{\sigma}} \\ \hat{\boldsymbol{\sigma}} & 0 \end{pmatrix}, \tag{4}$$

for the four-component wave function u. Here, $\hat{\beta}$ and $\hat{\boldsymbol{\alpha}}$ are the scalar and vectorial Dirac matrices determined through the four-component identity matrix $\hat{1}$ and the four-component vectorial Pauli matrix $\hat{\boldsymbol{\sigma}}$.

If we multiply this equation by the operator $\{E+e\varphi(r)-\hat{\beta} m_e c^2 - \hat{\boldsymbol{\alpha}} c \hat{\mathbf{p}}\}$ and use the operator identities

$$(\hat{\boldsymbol{\sigma}}\mathbf{A})(\hat{\boldsymbol{\sigma}}\mathbf{B}) = (\mathbf{AB}) + i\hat{\boldsymbol{\sigma}}[\mathbf{AB}], \quad (\hat{\boldsymbol{\sigma}}\mathbf{A})(\hat{\boldsymbol{\sigma}}\mathbf{A}) = \mathbf{A}^2 \tag{5}$$

and also the permutation relations for components of the vector Dirac matrix

$$\hat{\alpha}_i \hat{\alpha}_k + \hat{\alpha}_k \hat{\alpha}_i = 2\hat{\delta}_{ik}, \tag{6}$$

then we get a differential equation of the second order like the Schrödinger equation:

$$\{\hat{\mathbf{p}}^2/2m_e + U_{\text{eff}}\}u = \varepsilon u, \tag{7}$$

$$\varepsilon = [E^2 - (m_e c^2)^2]/2m_e c^2, \tag{8}$$

$$U_{\text{eff}} \equiv U_{\text{eff}}(r, E) = -e\varphi(r)(E/m_e c^2) - [e\varphi(r)]^2/2mc^2. \tag{9}$$

Here, $U_{\text{eff}}(r, E)$ is the effective potential energy determining the movement of an electron with full energy E in the field with potential $\varphi(r)$.

Equation 9 for the effective potential energy $U_{\text{eff}}(r, E)$ differs essentially from the "traditional" expression for interaction energy, $U_0(r) = -e\varphi(r)$, which is valid only in the nonrelativistic case (at $E \approx m_e c^2$ and $(E - m_e c^2)/m_e c^2 \ll 1$) and for comparatively weak fields ($|e\varphi(r)| \ll m_e c^2$).

In particular, for nonrelativistic particles with mass m and full energy $E \approx mc^2$ ($(E - mc^2)/mc^2 \ll 1$), the effective potential energy in a very strong field is determined by the first two terms in Eq. 9 and looks like

$$U_{\text{eff}} \approx U_{\text{eff}}(r, mc^2) \approx -e\varphi(r) - [e\varphi(r)]^2/2mc^2. \tag{10}$$

It follows from Eq. 10 that the effect of a strong scalar potential (at $|e\varphi(r)| > 2mc^2$) for nonrelativistic charged particles corresponds always to a nonlinear attraction ($U_{\text{eff}}(r, mc^2) < 0$) irrespective of the sign of a particle charge. This effect was discussed above in the analysis of the interaction of condensed pions with protons in a nucleus according to the Migdal model (see Ref. 4).

In the case of relativistic charged particles, except for a nonlinear attraction, the effective scalar potential can be increased due to the Lorentz transformation of a field with scalar potential $\varphi(r)$ for fast moving particles.

In view of the full energy of one relativistic electron,

$$\begin{aligned} E &= -e\varphi(r) + (p^2c^2 + m_e^2c^4)^{1/2} \\ &= -e\varphi(r) + T_1(p) + m_e c^2 \\ &\equiv -e\varphi(r) + \gamma_p m_e c^2, \end{aligned} \tag{11}$$

relation Eq. 9 allows one to get expressions for both the effective potential energy $U_{\text{eff}}(r, p)$ for an electron with momentum p and the similar effective energy of an electron $U_{\text{eff}}(r, p_F)$ on the surface of a Fermi sphere (with the limit momentum p_F)

$$\begin{aligned} U_{\text{eff}}(r, p) &= -e\varphi(r) - e\varphi(r)[T_1(p) - e\varphi(r)/2]/m_e c^2 \\ &= -e\varphi(r)\gamma_p + [e\varphi(r)]^2/2m_e c^2, \end{aligned} \tag{12}$$

$$\begin{aligned} U_{\text{eff}}(r, p_F) &= -e\varphi(r)E_F/m_e c^2 + [e\varphi(r)]^2/2m_e c^2 \\ &= -e\varphi(r)\gamma_F + [e\varphi(r)]^2/2m_e c^2. \end{aligned} \tag{13}$$

Here, $E_F = (p_F^2c^2 + m_e^2c^4)^{1/2} = \gamma_F m_e c^2$ is the Fermi energy, $T_1(p) = (p^2c^2 + m_e^2c^4)^{1/2} - m_e c^2 = (\gamma_p - 1)m_e c^2$ is the kinetic energy of one electron with momentum p in the degenerate relativistic electron gas, while γ_p and γ_F are, respectively, the relativistic Lorentz factors for an electron with momentum p and maximal momentum p_F on the surface of the Fermi sphere.

Let's note a basic difference of the expressions for the effective potential energy of an electron in the examined quasiatom compressed by external pressure and one in an ordinary atom held by the internal Coulomb interaction only.

In a bound electron-nuclear system without external influence (without additional external pressure), the full energy $E = -e\varphi(r) + T_1(p) + m_e c^2$ is always less than the rest energy $m_e c^2$. This follows, obviously, from the fact that the steady and bound state of an electron in an atom is impossible at $-e\varphi(r) + T_1(p) > 0$. Using the condition $E < m_e c^2$, it can be found from Eq. 10 that the maximal (by absolute value) potential energy of an electron in an ordinary atom is always limited

$$|U_{\text{eff(atom)}}(r, p_F)|_{(\text{max})} \leq e\varphi(r) + [e\varphi(r)]^2/2m_e c^2. \tag{14}$$

In the case of a compressed quasiatom, the full energy E of a single electron depends on external pressure which keeps the compressed quasiatom from dispersion and may increase its kinetic energy $T_1(p) = (\gamma_p - 1)m_e c^2$ unlimitedly, according to the unlimited increase of the ratio $E/m_e c^2$. It should be noted that an increase in the kinetic energy $\langle T_1(p)\rangle \gg \langle e\varphi(r)\rangle$ is possible under compression, which is unattainable, in principle, in the Coulomb field for which $\langle T_1(p)\rangle = \langle e\varphi(r)\rangle/2$ by the virial theorem.

Such a compression results in a drastic increase in the effective potential energy of an electron in the compressed quasiatom

$$|U_{\text{eff(quasiatom)}}(r, p)|_{(\text{max})} > e\varphi(r) + [e\varphi(r)]^2/2m_e c^2. \tag{15}$$

In particular, if $E \gg \langle e\varphi(r)\rangle$, then

$$|U_{\text{eff(quasiatom)}}(r, p)|_{(\text{max})} \gg e\varphi(r) + [e\varphi(r)]^2/2m_e c^2, \tag{16}$$

which corresponds to a very significant enhancement of the interaction of an electron with a nucleus. The physical reason for this is related to a relativistic transformation of a scalar potential in the case of moving particles.

This very fact allows us to realize the effect of self-compression of the electron-nuclear plasma considered below which may occur at a preliminary threshold compression of the substance. For "ordinary" atoms not compressed by external forces up to the state of degenerate relativistic gas, such an effect is impossible in principle.

Let's note one more important fact.

In the presence of other electrons in the volume of a Wigner–Seitz cell, the potential $\varphi(r)$ (and, consequently, effective potential U_{eff}) should be determined via the nucleus potential $\varphi_0(r) = Ze/r$ and the average potential formed by other electrons. The final concentration of electrons $n_e(r) = (1/3\pi^2\hbar^3c^3)[E_F - U_{\text{eff}}(r, p_F)]^3$ in the volume of a Wigner–Seitz cell is a result of such a self-coordinated interaction.

This problem may be solved by using the relativistic Thomas–Fermi equation

$$\Delta\varphi(r) = -4\pi e \left\{(1/3\pi^2\hbar^3c^3)[E_F - U_{\text{eff}}(r, p_F)]^3 - Z\delta(\mathbf{r})\right\} \qquad (17)$$

under specific boundary conditions.

This problem (for the boundary conditions $r\varphi(r) \to 0, r \to \infty$ and the simplified potential Eq. 10) was examined in Ref. 4 while analyzing the characteristics of a system of electrons in the field of a nucleus with charge Ze. It is obvious that such boundary conditions correspond to a free isolated atom not compressed by external forces. In the last case at $r \to \infty$, the potential energy $-e\varphi(r)$, the density of electrons $n_e(r)$, and the electron pressure P_e are equal to zero. The result is the presence of a shielded Coulomb potential in the volume of a "Thomas–Fermi atom" $\varphi(r) = \chi(r)Ze/r$, for which $\chi(r) = \exp(-r/R_{S(0)})$, $R_{S(0)} \approx 0.885(\hbar^2/m_e e^2)/Z^{1/3}$; $\chi(r) \to 1, r \to 0$; $\chi(r) \to 0, r \to \infty$. For such a "natural" isolated atom whose size and structure are determined only by the interaction between the nucleus and electrons, the self-compression is impossible.

Let's consider a basically different model: a compressed quasiatom whose size and structure are determined at the initial stage by external pressure.

Several works examined the characteristics of an extremely compressed degenerate electron gas surrounding the single nucleus. The screening distance $R_S = 5.8(3/4\pi n_e)^{1/3}$ for the nucleus in the infinite volume of a degenerate relativistic electron gas was calculated in Refs. 17, 18 by using the statistical dielectric function $\varepsilon_e(k, 0)$ (see Ref. 19). This result is not suitable for the following analysis of a compressed quasiatom with "ordinary" nucleus. It may be used only in the case where R_S is much less than the quasiatom radius: $R = Z^{1/3}(3/4\pi n_e)^{1/3}$ (i.e., $Z \gg 200$).

The model of compressed atom on the basis of the Thomas–Fermi equation was examined in Ref. 15. In particular, it follows from this model that, due to the increase in the concentrations of electrons and nuclei during compression, the role of the shielding effects becomes less substantial.

It is noted in Refs. 13–15 that, in the compressed environment with mass density $\rho \gg 10^4\,\text{g/cm}^3$ (it corresponds to the electron density $n_e \gg 3 \times 10^{27}\,\text{cm}^{-3}$) the state of a degenerate electron plasma corresponds to the ideal homogeneous gas with density n_e which is independent of coordinates. In this case, the Coulomb interaction may be taken into account as the interaction of this homogeneous gas with the field of a nucleus. The same result follows directly from Eq. 17 with R reduction.

These results simplify significantly the further calculations.

Consider a compressed quasiatom of radius R with the nucleus of charge Ze. The full Coulomb potential energy $U_{eQ} = U_{eQL} + U_{eQNL}$ of all Z degenerate electrons in the volume $V = 4\pi R^3/3$ consists from the linear (with respect to $e\varphi(r)$) part U_{eQL} and the nonlinear U_{eQN} (proportional to $(e\varphi(r)^2)$ one and may be deduced from Eq. (13) by summation of the contributions from all electrons according to their distribution within the Fermi sphere with the boundary (Fermi) momentum $p_F = (3\pi^2)^{1/3}\hbar n_e^{1/3}$.

The linear part of the potential energy is equal to

$$U_{eQL} = \sum_{i=1}^{Z} U_{\text{eff}}(r_i, p_i)$$
$$= \int_V \int_0^{p_F} U_{\text{eff}}(r,p)(d^2N/dpdV)dpdV = W_Q K_F, \qquad (18)$$
$$W_Q = -\int_V e\varphi(r) n_e dV, \qquad (19)$$
$$K_F \equiv \langle\gamma\rangle = (1/\pi^2\hbar^3 n_e m_e c^2)\int_0^{p_F} (p^2c^2 + m_e^2c^4)^{1/2} p^2 dp. \qquad (20)$$

Here, $d^2N/dpdV = p^2/\pi^2\hbar^3$ is the density of electron states in the r-space and p-space in the degenerate electron gas which is normalized by the full number of electrons:

$$\int_V \int_0^{p_F} (d^2N/dpdV)dpdV = Z. \qquad (21)$$

W_Q is the full Coulomb energy of electrons according to their homogeneous distribution in the volume of the compressed quasiatom but without regard for the influence of their movement in the Fermi condensate,
K_F is the factor of a dynamic increase in this energy related to the movement of electrons which is equal to the average value of the Lorentz factor $\langle\gamma\rangle$ for all electrons of the compressed quasiatom. For a relativistic degenerate gas of electrons, $\langle\gamma\rangle = (3/4)\gamma_F$.

The first part W_Q of the full energy U_{eQL} Eq. (18) is the sum of two components: the attraction energy of a nucleus and electrons:

$$W_{Q(en)} = \int_0^R V_0(r) n_e 4\pi r^2 dr = -(3/2)(Z^2e^2/R)$$
$$= -(3/2)(4\pi/3)^{1/3} Z^{5/3} e^2 n_e^{1/3} \quad (22)$$

and the repulsion energy of electrons:

$$W_{Q(ee)} = \int_0^R Q dQ(r)/r = (3/5)Z^2e^2/R$$
$$= (3/5)(4\pi/3)^{1/3} Z^{5/3} e^2 n_e^{1/3}. \quad (23)$$

Here, $Q(r) = -Zer^3/R^3$ is the total charge of all homogeneously distributed degenerate electrons which are in the sphere with radius $r < R$ inside the compressed quasiatom.

The factor of dynamic increase of the full Coulomb energy is equal to

$$K_F = (m_e^3c^3/8\pi^2\hbar^3 n_e)\{(p_F/m_ec)[2(p_F/m_ec)^2 + 1]$$
$$\times [(p_F/m_ec)^2 + 1]^{1/2} - Arsh(p_F/m_ec)\}. \quad (24)$$

In the case of a nonrelativistic degenerate gas (at $p_F \ll m_ec$ and $n_e < 10^{30}\,\mathrm{cm}^{-3}$),

$$K_F = 1 + (3/10)(3\pi^2)^{2/3}(\hbar/m_ec)^2 n_e^{2/3}$$
$$\approx 1 + 0.3(n_e/10^{30}\,\mathrm{cm}^{-3})^{2/3}. \quad (25)$$

In the case of superrelativistic gas (at $p_F \gg m_ec$ and $n_e \gg 10^{30}\,\mathrm{cm}^{-3}$),

$$K_F = (3/4)(3\pi^2)^{1/3}(\hbar/m_ec) n_e^{1/3}$$
$$\approx 0.77(n_e/10^{30}\,\mathrm{cm}^{-3})^{1/3}. \quad (26)$$

Accordingly, the nonlinear part of the full Coulomb potential energy reads

$$U_{eQNL} = \int_V [e\varphi(r)]^2 n_e dV/2m_ec^2$$
$$= \int_0^R (-Ze^2/r + Ze^2r^2/R^3)^2 n_e 4\pi r^2 dr/2m_ec^2$$
$$= [8\pi Z^{7/3} e^4 (3/4\pi)^{1/3}/7m_ec^2] n_e^{2/3}. \quad (27)$$

A change of the full Coulomb energy of the degenerate gas of electrons Eq. (22) and Eq. (23) is accompanied by a simultaneous change of the exchange energy of the degenerate electron gas (see Ref. 20)

$$U_{e,exch} = -(e^2/n_e)\int_0^{p_F} d^3p_1/(2\pi\hbar)^3 \times \int_0^{p_F} d^3p_2/(2\pi\hbar)^3 \frac{4\pi\hbar^2}{|\mathbf{p}_1 - \mathbf{p}_2|^2}$$
$$= -[3(3\pi^2)^{1/3}/4\pi]Ze^2n_e^{1/3} \tag{28}$$

and an increase in its kinetic (Fermi) energy

$$U_{eF} = \int_V \int_0^{p_F} T_1(p)(dN^2/dpdV)dpdV$$
$$= (V/\pi^2\hbar^3)\int_0^{p_F} [(p^2 + m_e^2c^2)^{1/2} - m_ec^2]p^2dp$$
$$= Zm_ec^2(K_F - 1). \tag{29}$$

The total energy of the degenerate electron gas is

$$U_{e\Sigma} = U_{eQL} + U_{eQNL} + U_{e,exch} + U_{eF}$$
$$= [W_{0(en)} + W_{0(ee)} + Zm_ec^2]K_F + U_{e,exch} - Zm_ec^2 + U_{eQNL}$$
$$= \{Zm_ec^2 - (9/10)(4\pi/3)^{1/3}Z^{5/3}e^2n_e^{1/3}\}K_F$$
$$- [3(3\pi^2)^{1/3}/4\pi]Ze^2n_e^{1/3} - Zm_ec^2$$
$$+ [8\pi Z^{7/3}e^4(3/4\pi)^{1/3}/7m_ec^2]n_e^{2/3}. \tag{30}$$

It is easy to see from these results that, for the relativistic degenerate gas, the nonlinear component of energy U_{eQNL} for a nucleus with $Z \ll 2800$ is much less than the linear component U_{eQL} and can be omitted for such nuclei.

2.3. Peculiarities of the Evolution and Collapse of a Neutral Wigner–Seitz Cell at High Densities of the Compressed Relativistic Degenerate Gas of Electrons

From Eq. (30) the effective total pressure of the electron gas on the quasiatom surface may be found as

$$P_e(Z, n_e) = -dU_{e\Sigma}/dV = (n_e^2/Z)dU_{e\Sigma}/dn_e. \tag{31}$$

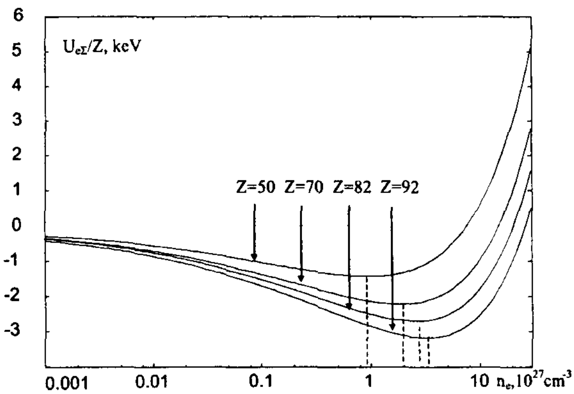

Fig. 3. Degenerate electron gas energy in the low-density area in the volume of a compressed stable quasiatom.

The dependence of the energy $U_{e\Sigma}$ Eq. (30) of the degenerate electron gas on its density n_e for nuclei with Z = 29, 50, 70, 82, 92 per electron is shown in Fig. 3... Fig. 6 for three ranges of the degenerate electron gas density n_e ($10^{24} \ldots 3 \times 10^{28}\,\mathrm{cm}^{-3}; 10^{29} \ldots 3 \times 10^{33}\,\mathrm{cm}^{-3}; 10^{32} \ldots 10^{35}\,\mathrm{cm}^{-3}$).

It follows from Fig. 3 that, at a rather low density of the nonrelativistic degenerate gas (in the range $n_e \approx (0.8 \ldots 3) \times 10^{27}\,\mathrm{cm}^{-3}$), a local minimum of energy corresponds to each type of nuclei. These minima correspond to zero pressure $P_e(Z, n_e) = 0$ on the surface of a compressed quasiatom. The condition $P_e(Z, n_e) = 0$ determines steady states of the compressed quasiatom which may exist without any external influence. They are analogs (in the area of high electron densities) of a metal bond in crystals which is realized in a degenerate nonrelativistic electron gas of conductivity electrons at their low density $n_e \approx 10^{23}\,\mathrm{cm}^{-3}$ in metals. It should be noted that, in the case of a neutral degenerate electron-ion plasma containing identical nuclei (Fig. 2), the same pressure will be applied to the external surface of this plasma.

Under a further compressing of the electron-nuclear plasma, the density n_e and energy of the electron gas increase, and the steady states are disrupted, which corresponds to the transition to an unstable plasma. For the confinement of such a plasma, it is necessary to apply an additional (negative) external pressure equal to the positive pressure on the surface of a compressed quasiatom $P_e(Z, n_e) > 0$. At a further increase in the density n_e (up to $n_{e(cr)} \approx 10^{32} \ldots 10^{33}\,\mathrm{cm}^{-3}$, which corresponds to a relativistic gas), the full energy of the compressed quasiatom reaches the maximum which corresponds to zero pressure $P_e(Z, n_{e(cr)}) = 0$ on the surface of a Wigner–Seitz cell and

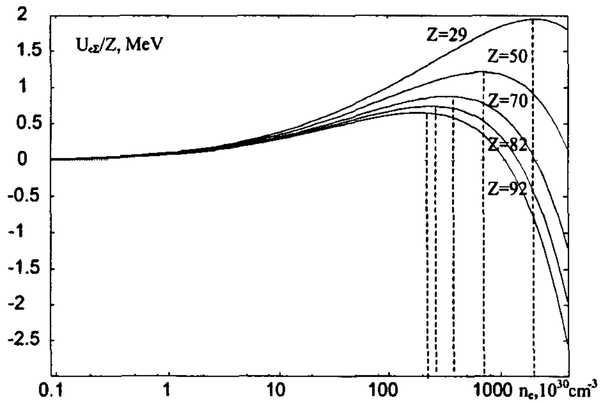

Fig. 4. Degenerate electron gas energy in the volume of a compressed stable quasiatom in the area of unstable balance.

to the state of unstable balance between the Fermi repulsion forces of electrons and the Coulomb attraction of electrons to a nucleus. The structure of the energy maximum is presented in Fig. 4 (for the average energy of each electron, $U_{e\Sigma}/Z$) and Fig. 5 (for the full energy of the degenerate gas, $U_{e\Sigma}$).

A further increase in the electron gas density reduces the full energy of the electron gas, which corresponds to the negative pressure $P_e(Z, n_e) < 0$ on the surface of a Wigner–Seitz cell. Pressure of this kind renders the compressed quasiatom at $n_e > n_{e(cr)}$ to be unstable relative to the spontaneous self-compression of the plasma and to the subsequent unlimited increase in the electron density n_e. In this case, the compression of the relativistic degenerate gas occurs without loss of its ideality. This self-reinforcing process of irreversible self-compression of the degenerate electron gas is accompanied by "the downfall of electron shells of the compressed quasiatom to the nucleus" and results in the full collapse of the electron-nuclear plasma in the volume of a particular Wigner–Seitz cell. The dependence of the average energy of the degenerate relativistic electron plasma in the stage of formation of a self-reinforcing collapse is shown in Fig. 6. It can be seen from this figure that the average binding energy of each electron may reach tens of MeV.

The threshold value $n_{e(cr)} \equiv n_{e(cr)}(Z)$, at which the pressure inversion and the formation of the collapse state take place, depends on the charge of a nucleus and monotonously increases with reduction in Z, starting from $n_{e(cr)} \approx 2 \times 10^{32}\,\mathrm{cm}^{-3}$ for the heaviest stable nucleus (a nucleus of uranium with $Z = 92$). For copper with $Z = 29$, $n_{e(cr)} \approx 2 \times 10^{33}\,\mathrm{cm}^{-3}$.

It seems at first sight that the necessary values $n_{e(cr)} \geq 2 \times$

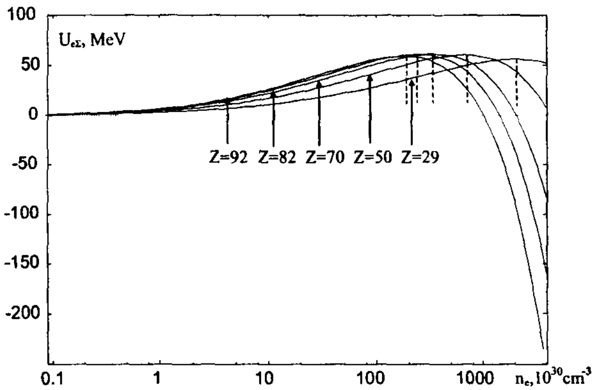

Fig. 5. Full energy of the degenerate superrelativistic electron gas in the volume of a compressed stable quasiatom in the area of unstable balance.

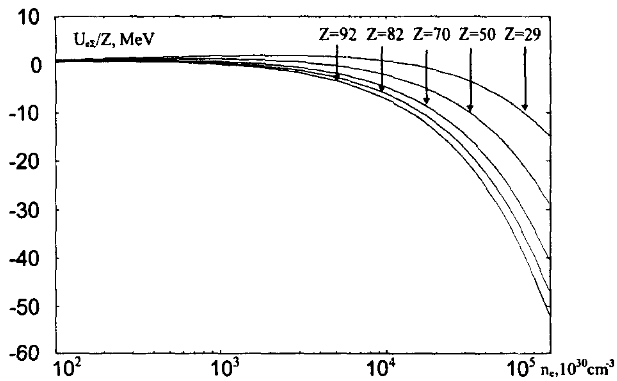

Fig. 6. Full energy of the degenerate superrelativistic electron gas in the volume of a compressed stable quasiatom in the stage of self-compression.

$10^{32}\,\mathrm{cm}^{-3}$ for the critical density of a degenerate electron gas are so great that they hardly can be reached. In fact, the situation is different.

The matter is that the defining parameter is not $n_{e(cr)}$, but the additional energy which is necessary for a preliminary compression of the electron-nuclear plasma up to this density.

It follows from the calculations carried out that a rather small energy is necessary for reaching the collapse threshold of a compressed quasiatom (the threshold density of electrons $n_{e(cr)}$). The average energy $U_{e\Sigma}/Z \approx 0.65\,\mathrm{MeV/electron}$ is necessary for a nucleus with $Z = 92$ for reaching the threshold density $n_{e(cr)} \approx 2 \times 10^{32}\,\mathrm{cm}^{-3}$. This value is much less than the kinetic (Fermi) energy

$$U_{eF}/Z = (3/4)(3\pi^2)^{1/3}\hbar c n_{e(cr)}^{1/3} \approx 3\,\mathrm{MeV/electron} \qquad (32)$$

required for the degenerate electron gas to reach $n_{e(cr)}$ without regard for the additional influence of the Coulomb attraction of nuclei. The difference of these values $U_{eF}/Z - U_{e\Sigma}/Z \approx 2.4\,\mathrm{MeV/electron}$ corresponds to a spontaneous increase of the binding energy of electrons with a nucleus in the volume of a compressed quasiatom. The physical explanation of such a favorable effect is related to the fact that, under an additional compression of the degenerate electron gas alongside with increase in its kinetic energy (which increases the full energy), the potential and full energies are reduced because, at such a compression, the electrons are located in the area of the stronger Coulomb field around a nucleus.

Thus, about 80 % of the activity necessary for reaching the collapse threshold is carried out by the electron-nuclear system on its own through the Coulomb interaction. Without this effect, at the expense of the energy of an external force $U_{e\Sigma}/Z \approx 0.65\,\mathrm{MeV/electron}$, it is possible to reach the degenerate electron gas density

$$n_{e(\mathrm{eff})} \approx [(64/81\pi^2)/(\hbar c)^3]U_{e\Sigma}/Z \approx 10^{30}\,\mathrm{cm}^{-3}, \qquad (33)$$

which is by 200 times less than that gained with the help of electron-nuclear Coulomb interaction.

Accordingly, the average energy $U_{e\Sigma}/Z \approx 0.75\,\mathrm{MeV/electron}$ is necessary for a nucleus of lead ($Z = 82$) to reach the critical density $n_{e(cr)} \approx 3 \times 10^{32}\,\mathrm{cm}^{-3}$. Without taking into account the Coulomb interaction, such an average energy allows one to reach the electron gas density

$$n_{e(\mathrm{eff})} \approx [(64/81\pi^2)/(\hbar c)^3]U_{e\Sigma}/Z \approx 1.2 \times 10^{30}\,\mathrm{cm}^{-3}. \qquad (34)$$

The dependence of the critical density on the charge of a nucleus results in the certain sequential stages of the transition of a nucleus to the state of collapse. Under a gradual increase in the external pressure, the first to pass to the state of collapse will be the electrons from the volumes of those Wigner–Seitz cells whose centers possess nuclei with the maximal charge. If the degenerate electron plasma contains different nuclei, then only those nuclei will pass to the state of electron-nuclear collapse which satisfy the condition of pressure inversion at the appropriate critical density $n_{e(cr)}(Z)$ in the volume. Thus, the relativistic degenerate electron-nuclear plasma may contain heavy nuclei in the state of electron-nuclear collapse as well as lighter nuclei which belong to compressed, but not collapsed quasiatoms.

3. SYNTHESIS OF SUPERHEAVY NUCLEI AND PECULIARITIES OF THE FORMATION OF A NUCLEAR CLUSTER

The process of formation of the electron-nuclear collapse begins after reaching the critical electron density $n_{e(cr)}$ and leads to an avalanche-type increase in the density of electrons $n_e \gg n_{e(cr)}$ in the degenerate relativistic gas up to the value comparable to the density of protons n_p in a nucleus. In such a process, there is a substantial increase in the binding energy of the electrons participating in the electron-nuclear collapse, which changes both the nature of intranuclear processes and the further evolution of the collapse.

First of all, it should be noted that the evolution of the collapse should be examined with regard for the full energy of a compressed quasiatom, including both the energy of a nucleus E_n Eq. (1) and the full energy of electrons $U_{e\Sigma}$ Eq. (30). In addition, the expression for the nucleus energy should be changed in view of the partial screening of the proton charge inside a nucleus by the degenerate gas of relativistic electrons, which is equivalent to a reduction of the charge of each proton ($e \to [1 - n_e/n_p]e$) or the number of protons ($Z \to [1 - n_e/n_p]Z$).

Figure 7 demonstrates the dependence of the full energy per nucleon of a compressed quasi-atom (Wigner–Seitz cells are in the phase of formation of the electron-nuclear collapse)

$$\begin{aligned}\Delta E_{en}/A = {} & \varepsilon_2/A^{1/3} + \varepsilon_3(1 - n_e/n_p)^2 Z_{\text{opt}}^2/A^{4/3} \\ & + \varepsilon_4[1/2 - (1 - n_e/n_p)Z_{\text{opt}}/A]^2 \\ & + \varepsilon_5/A^{7/4} + U_{e\Sigma}(Z_{\text{opt}})/A \end{aligned} \tag{35}$$

on both A and the degenerate relativistic electron gas density in the range $1 < A < 700$.

In calculations, we used the classical line of stability $Z_{\text{opt}}(A)$ for the charge of a nucleus by taking the screening into account:

$$\begin{aligned}Z_{\text{opt}} &= \frac{A}{2[1 + (\varepsilon_3/\varepsilon_4)(1 - n_e/n_p)^2 A^{2/3}]} \\ &\approx \frac{A}{2 + 0.0155(1 - n_e/n_p)^2 A^{2/3}}.\end{aligned} \tag{36}$$

For a convenience of the analysis, the value $\Delta E_{en}/A$ was normalized to the energy of the most stable nuclei such as iron, the constant total energy of the rest of all nucleons and electrons, and the binding energy of all nucleons $\varepsilon_1 A$.

The results of calculations yield that, by increasing the density of electrons in the volume of a nucleus, the energy of the electron-nuclear

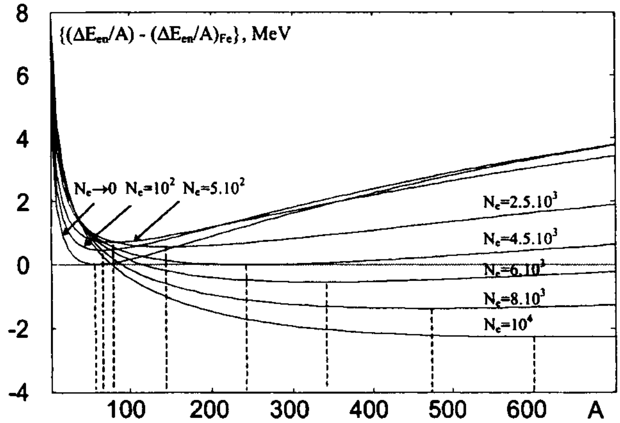

Fig. 7. Energy of a compressed quasiatom in the state of partial collapse of the electron-nuclear degenerate plasma for different concentrations of the degenerate superrelativistic electron gas $N_e = (n_e/10^{30}\ \mathrm{cm}^{-3})$.

system continuously reduces (this is caused by the corresponding increase of its binding energy), and the minimum of this energy shifts to the area of big masses A_m.

At $n_e < 10^{30}\,\mathrm{cm}^{-3}$ ($N_e \equiv n_e/10^{30}\,\mathrm{cm}^{-3} < 1$), the minimum of the electron-nuclear system energy corresponds to nuclei with mass number $A_{\mathrm{opt}} \approx 60$. With increase in the electron density, the minimum of the full energy A_{opt} increases as well (see Table 1).

It is obvious that, at reaching the density $n_e > 10^{35}\,\mathrm{cm}^{-3}$, the minimum of the electron-nuclear system energy shifts to $A_{\mathrm{opt}} > 3000$, where the threshold state for the existence of a nucleus with pion condensate becomes possible.

The position shift of A_{opt} for a minimum of the full energy E_{en}/A is accompanied by a change of the energy of this minimum from the initial value $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} = 0$ at $n_e \ll 10^{30}\,\mathrm{cm}^{-3}$ (this corresponds to nuclei of the iron group) at first to the side of increasing the minimum up to $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} \approx 0.65\,\mathrm{MeV/nucleon}$ at $n_e = 2.5 \times 10^{32}\,\mathrm{cm}^{-3}$. Then its irreversible reduction occurs: $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} \approx 0$ at $n_e = 4.5 \times 10^{33}\,\mathrm{cm}^{-3}$, $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} \approx -1.4\,\mathrm{MeV/nucleon}$ at $n_e = 8 \times 10^{33}\,\mathrm{cm}^{-3}$, $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} \approx -2.2\,\mathrm{MeV/nucleon}$ at $n_e = 10^{34}\,\mathrm{cm}^{-3}$.

The derived values of A_{opt} and $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}}$ characterize the direct influence of degenerate electrons (owing to the self-energy of the electron condensate).

Fig. 8 shows the dependence of the same parameter $\Delta E_{en}/A$ in a greater range of A and n_e. It is seen that, with increase in the electron density up to $n_e \approx 10^{35}\,\mathrm{cm}^{-3}$, this minimum reduces down to $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} \approx -30\,\mathrm{MeV/nucleon}$.

Table 1. Atomic mass $A_{\rm opt}$ corresponding to the minimum of the full energy of the electron-nuclear system with electron density n_e

n_e, $\rm cm^{-3}$	$A_{\rm opt}$
$< 10^{30}$	≈ 60
$< 10^{32}$	≈ 70
$< 5 \times 10^{32}$	≈ 80
$< 2.5 \times 10^{33}$	≈ 150
$< 4.5 \times 10^{33}$	≈ 240
$< 6 \times 10^{33}$	≈ 340
$< 8 \times 10^{33}$	≈ 470
$< 10^{34}$	≈ 600
$< 10^{35}$	≈ 2000

Such a shift of the position and depth of the energy minimum of the electron-nuclear system opens a way to the formation of extremely heavy nuclei through the repeated absorption of "ordinary" nuclei by the nucleus in a state of collapse.

It is necessary to note that even greater shifts of a minimum of the full energy $A_{\rm opt}$ and $\Delta E_{en}(A_{\rm opt})/A_{\rm opt}$ are possible. The reason for this is that the classical line of stability $Z_{\rm opt}(A)$ for the charge of a nucleus Eq. 36 corresponds to the optimization condition of nuclear parameters on the basis of the formula $d(E_n/A)/dZ = 0$. If the optimization involves both nuclear and electron parameters with regard for the condition $d(E_e n/A)/dZ = 0$, then we can find the corrected value $Z_{\rm opt}(A, n_e)$ which corresponds to a deeper minimum of the electron-nuclear system energy.

For example, at the density of electrons $n_e \ll 10^{42}/A^3 Z^2\,\rm cm^{-3}$, the optimum charge of a nucleus (and the number of electrons in a compressed quasiatom) is determined by

$$Z_{\rm opt}(A, n_e) = \frac{A[1 + ABZ^{2/3} n_e^{1/3}]}{2[1 + (\varepsilon_3/\varepsilon_4)(1 - n_e/n_p)^2 A^{2/3}]}\,, \qquad (37)$$

$$B = (9/10)(4\pi/3)^{1/3} e^2/\varepsilon_4 \approx 10^{-14}\ \rm cm. \qquad (38)$$

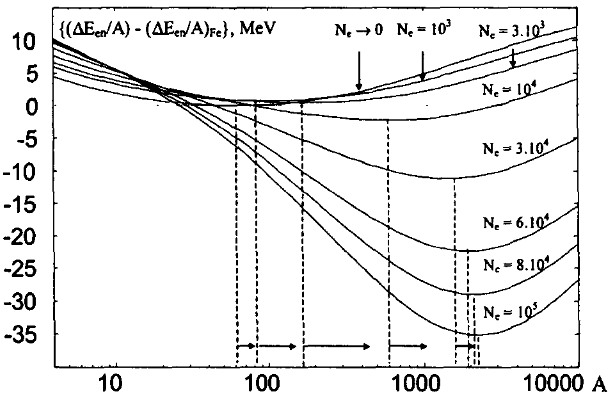

Fig. 8. Energy of a compressed quasiatom in the state of partial collapse of the electron-nuclear degenerate plasma for different concentrations of the degenerate superrelativistic electron gas $N_e = (n_e/10^{30}\ \mathrm{cm}^{-3})$. Arrows in the lower part of the figure show the direction of a shift of the full energy minimum (or binding energy maximum) of a compressed quasiatom.

The synthesis of superheavy nuclei is facilitated also by the fact that the electric field of a nucleus on the boundary of a compressed quasiatom (on the boundary of a Wigner–Seitz cell) is completely neutralized by the condensed electrons. The reduction of the compressed quasiatom radius in its collapse greatly reduces the influence of the Coulomb barrier on the synthesis of nuclei.

By a moderately strong compression of the degenerate relativistic gas of electrons (up to the density $n_{e(cr)} \approx 10^{32}\,\mathrm{cm}^{-3} < n_e \ll n_p \approx 10^{38}\,\mathrm{cm}^{-3}$ which is much less than the density of protons n_p), the width of the Coulomb barrier is strongly decreased: from the initial value equal to the screening radius $R_{S(0)} \approx 0.885(\hbar^2/m_e e^2)/Z^{1/3}$ in atoms and ions with a small degree of ionization or in the uncompressed degenerate electron plasma up to $R_S \approx R = (3Z/4\pi n_e)^{1/3}$ determined by the radius R of a Wigner–Seitz cell. In particular, for a nucleus with $Z = 92$, the initial screening radius $R_{S(0)} \approx 0.1$ Å. The Coulomb barrier width decreases at the density $n_{e(cr)} \approx 10^{32}\,\mathrm{cm}^{-3}$ by $R_{S(0)}/R_S \approx 16.6$ times, at the density $n_{e(cr)} \approx 10^{33}\,\mathrm{cm}^{-3}$ by $R_{S(0)}/R_S \approx 36$ times, at the density $n_{e(cr)} \approx 10^{35}\,\mathrm{cm}^{-3}$ by $R_{S(0)}/R_S \approx 166$ times, and at the density $n_{e(cr)} \approx 10^{37}\,\mathrm{cm}^{-3}$ by $R_{S(0)}/R_S \approx 775$ times. As the probability of a tunnelling transition of a nucleus through the Coulomb barrier in the compressed electron-nuclear plasma depends basically on the barrier width, then such a drastic reduction of the barrier width increases greatly the probability of the synthesis.

There are other estimations proving the efficiency of the synthesis.

It is possible to characterize the nuclei of a cold compressed electron-nuclear plasma by the effective energy of relative movement T_{eff} which should be possessed by the similar, but free nuclei in order that they can approach one another to the same distance R_S. For a pure Coulomb nonscreened interaction of two nuclei with charges Z_1 and Z_2 (it corresponds to the examined case of relativistic degenerate electron gas), this energy is determined by $T_{\text{eff}} = Z_1 Z_2 e^2/R_S$. For example, in order that a free nucleus of lead and a proton approach one another to the the distance $R_{S(0)} \approx 1.2 \times 10^{-12}$ cm (it corresponds to the density $n_e \approx 10^{36}\,\text{cm}^{-3}$), the relative energy $T_{\text{eff}} \approx 15$ MeV (or 15 MeV/nucleon) is required. Accordingly, the interaction of two lead nuclei in a degenerate electron gas with the same density requires the energy $T_{\text{eff}} \approx 1400$ MeV (or the same 15 MeV/nucleon). With increase in the density of electrons, T_{eff} grows. Such an energy renders the probability of the tunnelling effect to be high.

Such a synthesis has one additional effect.

In the process of such a fusion, the excessive energy is released via different channels such as emission of gamma quanta, neutrons, nuclear fragments, etc. One of the channels is connected with the creation of different "ordinary" nuclei and the emission of these nuclei from the volume of a growing superheavy nucleus. For example, after the absorption of several nuclei with $A_T \approx 50\ldots200$ in a short time, high binding energy can lead to the emission of several light nuclei with $A_L < A_T$ or one heavy nucleus with $A_H \approx 300\ldots500 > A_T$.

Such a process of nuclei emission competes with other channels of cooling of the nuclear substance. In this case, even-even nuclei (He^4, C^{12}, O^{16}, …, Pb^{208}) which already exist in the volume of a superheavy nucleus are more likely to emerge and be emitted. In fact, every superheavy nucleus in the phase of formation of an electron-nuclear collapse may be a "specific microreactor" for the transmutation of "ordinary" nuclei to different configurations of nucleons. In this microreactor, the process of transmutation comes to the end after the utilization of all neighboring nuclei or after the evolution of a superheavy nucleus to the final stable state with A_{max}.

In conclusion, it should be noted that the problem of self-compression of the system of degenerate relativistic electrons being in a compressed quasiatom (i.e., the achievement of the maximal ratio n_e/n_p and the weakening of interactions between protons inside a nucleus) may be solved only by performong the full analysis of the whole variety of electron-nuclear transformations (including the processes of protonization and neutronization and the formation of electron-positron pairs) for a specific nucleus. At the same time, it is clear that the boundary density of the compressed electron gas $n_{e(max)}$ will always be less than the density of protons in a nucleus n_p because the only reason for the compression of the relativistic degenerate gas of electrons is its interaction with the effective charge of a nucleus which decreases in the process of compression $Z^* = Z(n_p - n_e)/n_p$.

4. SUMMARY

We have proposed a theoretical model of a threshold-type self-similar electron-nuclear collapse gained at reaching the energy density in the electron plasma of about 0.65 MeV/electron, which corresponds to the high critical density of the degenerate electron gas $n_{e(cr)}$. This density is very high as compared with the value of $n_e \approx 10^{24}\,\mathrm{cm}^{-3}$ typical, for example, of the natural state of the conductivity zone in metals. At the same time, the value $n_{e(cr)}$ appears to be less by many orders of magnitude than the final density $n_e \approx 10^{36} \ldots 10^{37}\,\mathrm{cm}^{-3}$ which is automatically reached after a preliminary compression in the self-organizing process of formation of the collapse state and is comparable to the intranuclear density of nucleons n_n. In the collapse state, the direction and probability of nuclear reactions are changed significantly, the influence of the Coulomb barrier is greatly reduced, and the spontaneous synthesis of superheavy stable nuclei becomes probable. This approach differs basically from the "force" method of nucleosynthesis.

It is very important that the formation of the state of electron-nuclear collapse of a nucleus is based on its own electron system, thus, this process is adapted to this nucleus in the best possible way and is a coherent adiabatic process.

The consequences of such a formation of the the state of collapse are diverse and evident.

A part of them is connected with the possibility of the synthesis of superheavy nuclei with masses exceeding those of transuranium isotopes by many times. Such a synthesis becomes possible because the energy minimum of the system of electrons and nuclei shifts during the fast formation of the state of collapse from the area of mass numbers $A_{\mathrm{opt}} \approx 60$ (it corresponds to ordinary nuclei and a low density of electrons) to $A_{\mathrm{opt}} > 2000 \ldots 3000$ (at $n_e \approx 10^{35} \ldots 10^{36}\,\mathrm{cm}^{-3}$). This shift ensures the energy-efficient synthesis by nuclei absorption and creates the basic prerequisite for the formation of stable superheavy nuclei. The high efficiency of the synthesis is determined by the fact that, for a nucleus in the state of electron-nuclear collapse, the Coulomb repulsion appears to be greatly weakened because of the strong screening of a nucleus field by the degenerate compressed relativistic electron gas. This fact may result in a specific selection of the nuclei which are characterized by a high probability of interaction. On the one hand, the starting process of formation of state of collapse is greatly facilitated for a heavy initial nucleus with high charge Z. On the other hand, the interaction and the subsequent synthesis of a nucleus with a high charge Z is significantly complicated by the residual Coulomb repulsion. As a result, the optimum can be reached with a small quantity of initial heavy nuclei in a target which are diluted by a plenty of very light nuclei. In such an optimum situation, each type of nuclei performs a strictly defined function: on the basis of the heavy nuclei, the highly compressed electron-nuclear systems are formed, such as Wigner–Seitz cells in the state of collapse; whereas the light nuclei whose charges are insufficient for the formation of the state of collapse are efficiently absorbed by the compressed electron-nuclear systems, by increasing their mass and performing the role of a specific "building material".

Within the area of the formed electron-nuclear collapse, the interaction of "normal" nuclei with compressed quasiatoms may release the very high binding energy. Thereof, the resulting pattern is very complicated including the combinations of numerous acts of synthesis and decay with the formation of new nuclei and the accompanying proton-neutron transmutations, although the general trend is the evolution toward the energy minimum at A_{opt}.

In particular, the high binding energy released in each act of absorption of one or several heavy nuclei by compressed quasiatoms in the process of formation of the electron-nuclear collapse, can ensure the simultaneous emission of several lighter nuclei or the creation of one superheavy nucleus. Such a mechanism of transformation of the target nuclei to another nuclei was considered for the first time in Ref. 6. Apparently, the synthesis of superheavy nuclei and also of "normal" elements with anomalous isotope ratio, which was experimentally observed as a result of the pulse coherent driver impact on chemically pure targets (see Ref. 6–9), is related to such processes. Our estimations showed that the parameters of the coherent driver in those experiments allowed us to reach the specific energy of the compressed degenerate electron plasma on the level of 0.7 . . . 1 MeV/electron, which ensured the critical density of the degenerate gas of electrons $n_{e(cr)}$ for several heavy nuclei. There are convincing evidences for that the electron-nuclear plasma collapse was reached in the above-mentioned experiments.

The obvious result of the process of formation of the state of electron-nuclear collapse is that it may actually ensure the transition from the area of "normal" nuclei to the area of such superheavy nuclei where the stability of nuclear matter is provided by the pion condensate.

Probably, when the electron-nuclear system under study reaches the critical electron density $n_{e(cr)}(Z)$, the shift effect of the minimum of its full energy, ΔE_{en}, may be the key mechanism, through which it is possible to perform both the direct synthesis of superheavy nuclei with charges within area 2 with $120 < Z < 1600$ (Fig. 1), as well as the fast transition from area 1 of stable "normal" nuclei with $A < 240$, $Z \leq 92$ to area 3 of nuclei with the self-formed pion condensate $Z > 1600$

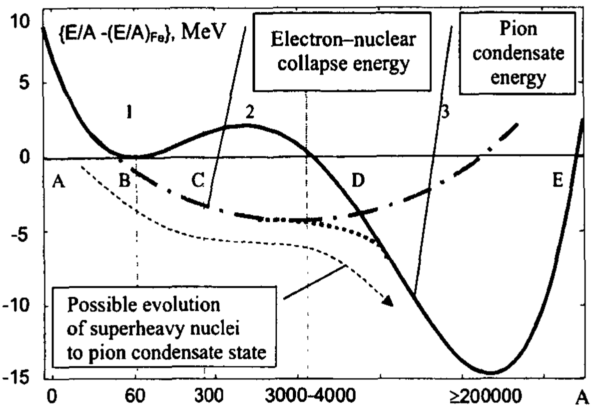

Fig. 9. Probable scheme of a nucleus energy behavior at normal electron and nuclear densities (upper curve in area ABC), with electron-nuclear collapse (lower curve in area BCD), and with pion condensate in a nucleus (lower curve in area DE). The arrow indicates the possible evolution of superheavy nuclei to the pion condensate state.

(Fig. 9), ensuring the conditions for the formation of a pion condensate at $A \rightarrow A_{\mathrm{opt}} \geq 200\,000$.

In conclusion, we note that the problems related to the nature and direction of nuclear reactions in the zone of electron-nuclear collapse are more complicated in comparison to the similar problems concerning with ordinary nuclei, because the former involve the processes of protonization and neutronization and the processes of absorption and formation of nuclear fragments. Very close to this circle of problems is the problem of self-compression of the system of degenerate relativistic electrons being in a compressed quasiatom.

The problem of stability of superheavy nuclei, which are created during the collapse of the electron-nuclear system including nuclei and the Fermi degenerate electron condensate, has to be analyzed with regard for the whole range of nuclear transformations occurring in the volume of synthesis (including the competitive processes of nuclear decay, nuclear beta-processes, and pion condensation). Such a task is exceptionally complicated and is a subject of future publications.

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