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**Distributed Bosonic States and Condensed
Matter Fusion**

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<p>We explain how it is possible for deuterons separated by macroscopic distances to interact in a nuclear fashion through the formation of a Bose Bloch Condensate (BBC) within a solid. Under suitable conditions, the formation of a BBC may lead to nuclear fusion and a variety of heretofore unobserved nuclear processes. We examine the application of these ideas to the anomalous heating of Pd through the electrolysis of D₂O and LiOD and conclude that only a small concentration of BBC deuterons is required. We also suggest various experiments associated with condensed matter fusion that may provide a test of our theory.</p>					
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DISTRIBUTED BOSONIC STATES AND CONDENSED MATTER FUSION

INTRODUCTION

This paper presents a theory of condensed matter fusion¹ in crystals based on the periodic potential provided by an ordered lattice and the approximate Bose statistics obeyed by bosons on length scales associated with electrostatic interactions. The resulting nuclear interaction proceeds in a manner that approximately preserves either the initial periodicity of the lattice or the periodicity of a commensurate lattice. Reaction reversibility over timescales associated with electrostatic interaction results in condensed matter fusion being dominated by interactions in which both the products and reactants obey Bose-Einstein statistics. As a result tritium release and $^3\text{He}^{++}$ production should not appear as primary fusion products.

BOSE BLOCH CONDENSATE

The key attribute of Bose particles needed is the property that the many body boson wave function possesses positive parity with respect to exchange of single particle wave functions. As a result any number of bosons possessing a given energy can occupy a particular lattice site (which is not the case for fermions). Huge degeneracy can occur within the many body boson wave function. A condition for fusion is that a portion of the D atoms (and possibly other atoms and ions) that enter the electrode reside in a screened ion form and see a periodic potential. Such ions can form a new bosonic state, which we call a boson Bloch condensate (BBC). The BBC is a result of the lattice dependent interplay of the symmetries associated with periodicity and the huge degeneracy of the boson wavefunction.

In accord with Bloch's theorem², the eigenstates of the crystal potential can be written in terms of single particle wave functions, ψ_1 , which have an associated wave-vector \underline{k} , such that

$$\psi_1(\underline{r}+\underline{R}_n) = \exp(i\underline{k}\cdot\underline{R}_n) \psi_1(\underline{r}) . \quad (1)$$

Index i designates the eigenvalue ϵ_i associated with wave vector \underline{k} (i.e., $\epsilon_i = \epsilon_i(\underline{k}) =$ the i^{th} "bosonic energy band" evaluated with wave vector \underline{k}). R_n is a Bravais lattice vector. The equation need not be restricted to three dimensions: for the desired overlap to occur, the periodicity of the lattice may be one-, two-, or three- dimensional. Equation (1) leads to the obvious relationship,

$$\psi_j(r) \psi_i(r+R_n) = \exp(i\underline{k} \cdot R_n) \psi_j(r) \psi_i(r) , \quad (2)$$

where $\psi_j(r)$ and $\psi_i(r)$ are separate initial state single particle wave functions.

Equation (2), though seemingly trivial, has important physical consequences. The equation implies that even at macroscopically large distances the overlap between boson single particle wave functions is order unity. This overlap is astronomically large in comparison with the overlap between charged bosons in free-space, where even at molecular length scales, the exponential damping associated with negative values of the kinetic energy leads to entirely negligible wavefunction value. Because the left side of Eq. (2) is one of many equivalent single site contributions to the many-body initial state wave function (which involves comparable products throughout the lattice), it follows that the fusion rate for bosons within the lattice should be very different than in free space. This fusion rate is proportional to the square of a matrix element derived from the integral of a nuclear potential (associated with the strong forces that bind nucleons inside the nucleus) ΔV_{strong} multiplied by the product of suitable initial and final states. Because the bosons are indistinguishable, they behave as a coherent entity and a remarkable effect becomes possible: when each ion responds to changes in its own local potential, it automatically induces changes in all of the potentials associated with the remaining indistinguishable ions. Thus, through "self-interaction" (the process in which an ion sees its own potential and readjusts its motion and spatial distribution, thereby modifying its own potential) all of the ions collectively interact with each other. Then, under suitable conditions (as discussed below), they can participate collectively in nuclear interactions at isolated locations, circumventing the formidable tunnelling problem of free space nuclear physics.

Because the BBC is composed of bosons, ϵ_i may equal ϵ_j in Eq. (2). The resulting possibility of huge degeneracies associated with the many body boson wave function has important consequences. In particular, formation of a D^+ BBC is a requirement for condensed matter fusion. Such a condensate resembles a quantum fluid, such as HeII, in being composed of indistinguishable particles. However, the BBC is a distinct matter state, differing in important ways from such quantum fluids. Unlike HeII, BBC requires a host lattice. Also, it can exist at very low boson concentration \underline{c} (provided the number of bosons N_b is macroscopic) and does not form exclusively through the ground state but possesses many forms (or phases) associated with the occupation of different quasi-continuous energy bands. Thus, it may exist at elevated temperatures. Also, the BBC couples preferentially to final states in which there are no isolated fermions. This leads to the selection rule of BBC reactions: "bosons in and bosons out". Also, because the BBC is composed of bosons, at low temperatures, collectively all of its constituents will occupy the lowest $k=0$ energy band, and a "true" Bose condensate will form, which will exhibit ionically mediated superconductivity and the Meissner effect.

The host lattice determines the spatial extent of the single body boson wave function. The lattice provides periodic potential wells in which D^+ ions reside. The magnitude of the zero point motion $r_{z.p.}$ associated with the classical turning points of each potential well defines the minimum "zero point volume" $V_{z.p.} = 4\pi/3r_{z.p.}^3$ occupied by a single boson. As a consequence, the maximum charge density of each BBC D^+ ion is greatly reduced from its peak free space particle value. We will show that the value of the lower bound of \underline{c} that is needed to produce a fusion power density of 10 W/cm^3 is about 10^{-7} D^+ per host atom. Since this value is much less than the D concentration used in fusion experiments¹, our model predicts that most of the D content is not part of the BBC.

A key property of a BBC is its ability to form multiple occupancy band states. Consider the idealization of a group of bosons which do not interact with each other but do interact with the lattice. These bosons distribute themselves throughout the lattice as a perfect gas. Their distribution includes the multiple occupancy of a unit cell. If the concentration of these

bosons is c_B , the concentration of n -boson unit cell occupancies is c_B^n in the low concentration limit. These multiple occupancies manifest themselves in the many body wave function as products of Bloch state wave functions of the form $\prod \psi_i^{\text{Bloch}}$, which may be viewed as a multiple-particle generalization of a band state. The "4-particle band" is the probable reactive component in condensed matter fusion.

DYNAMICS OF BBC SELF-INTERACTION

In a BBC, there are no individual particles. There are only collective states. These collective states have properties common to all quantum fields and are subject to self-fluctuation. Because of the indistinguishability of the bosons in the absence of outside perturbations, we have two choices: either 1) an individual boson at a given lattice site can be viewed as a composite occupation of many single particle eigenfunctions, where each eigenfunction contributes a fraction of the over-all charge (and wave function amplitude) and each is normalized to the volume of the entire crystal, or 2) each function can be normalized to a single unit cell whose identity with respect to other unit cells within the crystal is lost. The second option provides a method for identifying individual virtual bosons within a unit cell. In either case, once a perturbation that disrupts the periodicity of the potential is introduced at an isolated site, a proper accounting of the associated many body interactions throughout the crystal requires that the initial many body BBC state be viewed in the framework of a condensate, made up of the products of 1, 2, 3, 4, ..., N_B single particle Bloch state wave functions.

The second option provides an alternative to treating the lattice as a whole. It provides a picture in which many "particle-like" objects, such as di-particles (involving the product of two single particle wave functions), may participate in a disruption of periodic order at a specific site. The "multiple-particle bands" that correspond to these multiple-particle-like objects have equal density at each lattice site. In the absence of self-interaction, the "multiple-particle bands" have the same energy as the sum of the individual "single-particle band" energies. Each particle-like object

(single or multiple) may be expressed in terms of sums of products of Wannier states (ϕ_n 's), where the density $|\phi_n|^2$ describes the transient integer occupancy at the n^{th} lattice site. (Each of these is a virtual state.) The virtual single occupancy state has a self-interaction energy approximately equal to the energy of a real D^+ ion in the host lattice since the BBC is in thermal equilibrium with real interstitial D^+ . The multiple n -occupancy virtual states have a higher electrostatic self-interaction energy $\Delta E^{(n)}$ calculable from their overlapping charge distributions at a site. For the double occupancy case, corresponding to the difference in self-interaction energies between a doubly and singly charged particle,

$$\Delta E^{(2)} = 3/2 e^2 \int \int \frac{\rho(r) \rho(r')}{|r-r'|} dr dr', \quad (3)$$

where $\rho(r)$ is the particle density of the two particle Wannier state. We use a Gaussian to approximate the ground state spatial distribution of each Wannier state density,

$$|\phi_n|^2 = (2/\pi)^{3/2} \exp(-2|r-R_n|^2/r_{z.p.}^2)/r_{z.p.}^3, \quad (4)$$

about the the lattice site R_n . For a typical value of $r_{z.p.} = 0.5$ bohr, $\Delta E^{(2)} = 81.6$ eV and $\Delta E^{(4)} = 408$ eV. The quad-deuteron value is very much less than the self-interaction energy ($\Delta E^{(4)}_{\text{free}}$) of 4 free-space deuterons located at a common origin. $\Delta E^{(4)}_{\text{free}}$ is tens of millions of eV since the electrostatic self-interaction energy is comparable to the nuclear self-interaction energy $\Delta E^{(4)}_{\text{nuc}}$. Thus, the lattice-induced broadening of the charge distribution reduces ΔE^4 so strongly that electrostatic effects may be treated as a small perturbation relative to those due to $\Delta E^{(4)}_{\text{nuc}}$. As a consequence, it is possible to isolate the effects of electrostatic self-interaction from those due to changes in the strong potential ΔV_{strong} through a decoupling of the center of mass motion from the motion associated with variations in proton-neutron separation. A similar decomposition is not possible in free space where both forms of self-interaction proceed at comparable rates. The electrostatic lifetime $\tau^{(n)}$ of the virtual states is determined by Planck relationship $\tau^{(n)} = h/\Delta E^{(n)}$, where h is Planck's constant. For the di- and quad- occupancy states, $\tau^{(2)} = 5.1 \times 10^{-17}$

s, and $\tau^{(4)} = 1.0 \times 10^{-17}$ s, which should be compared with a nuclear Planck lifetime $\tau_{nuc} \sim 10^{-22}$ s. The large difference in lifetime means that an equilibrium condition can exist between a spatially compact nuclear state involving four deuterons and the four particle virtual state over a timescale that is long with respect to the nuclear interaction time but short with respect to self-induced electrostatic fluctuation. Thus, it becomes possible for all of the ions to participate collectively in nuclear interactions at isolated locations.

FUSION FROM THE BBC

Let us now consider the fusion problem. The most probable reaction appears to be the pure bosonic, momentum balanced reaction



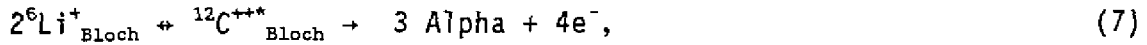
or its Wannier state equivalent,



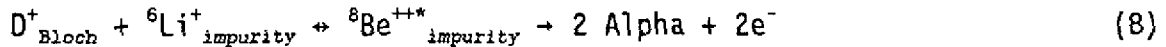
In Eqs. (5) and (6), $4D^+$ refers to the 4-fold occupancy state or quad-deuteron, and ${}^8Be^{++++*}$ stands for a compact configuration at nuclear density which possesses the symmetries and internal quantum numbers associated with the initial four deuterons. This intermediate state has the same total energy as the initial four deuterons, but nucleons within the "nuclear force bag" have high kinetic energies. The final state requires rearrangement of nucleons so as to produce two spin=0 Alpha's each containing 23.8 MeV relative to the center of mass. The full reaction is in principle reversible and satisfies the "bosons in and bosons out" selection rule (all components can exist as BBC states).

We also consider the possibility of two other reactions, suggested by the apparent requirement that LiOD (as opposed to NaOD) be electrolyzed to produce anomalous heat in a Pd electrode. With overvoltage electrolysis some Li can be expected to enter the host lattice. Although the effect of interstitial Li

is likely an electronic structure effect, there could be possible nuclear reactions:



and



In Eq. (7), 2^6Li^+ is a double occupancy $^6\text{Li}^+$ Bloch state. In Eq. (8), $^6\text{Li}^+_{\text{impurity}}$ is a real interstitial Li^+ ion which is assumed to react with the D^+ BBC initial state. Equation (8) reflects an apparently different type of BBC reaction. However, since the final Alpha particle states can be considered impurity states, the difference may be more apparent than real. We will not further consider either Li reaction.

Let us now consider whether the formation of a D^+ BBC can lead to the observed heat release in Pd by fusion reaction 5. Details of our analysis are given in the Appendix and elsewhere³. Using the observed¹ value of the power density $P_{\text{fusion}} \sim 10 \text{ W/cm}^3$ (4 MJ/120 hours), we set an approximate lower bound ($\underline{c}^{\text{lower}}$) for the concentration (\underline{c}) of injected D^+ ions, required to obtain the observed¹ heating. Our estimate of $\underline{c}^{\text{lower}}$ follows from an estimate of the fusion rate, Γ_{fusion} , based on the Fermi Golden rule.

Because the self-induced perturbation, ΔV_{strong} , associated with Eq. (5) is short-ranged and involves interactions between eight nucleons, two consequences follow from Eq. (1). 1) Large contributions to the associated perturbation matrix element result from the product between any four single particle D^+ Bloch states throughout the lattice in the initial state and the product of the highly localized perturbation with the final state ψ_{α} functions associated with the recoiling alpha particles. 2) All contributions to the matrix element are negligible except for those associated with the integrations over regions near the site where interaction takes place. Then, the number of distinct contributions that arise from the overlap of each of the N_B ($\gg 1$) single particle Bloch states associated with the initial state (in which there

is at most 1 D^+ wave function centered at each site) with the product of ΔV_{strong} and the 4 D^+ -double-alpha particle final state wave functions near the location R_{fusion} of the perturbation is $N_b(N_b-1)(N_b-2)(N_b-3) = N_b^4$. Hence, given an approximate value for the energy release per fusion, Q_{fusion} , for fixed values of $P_{\text{fusion}} = \Gamma_{\text{fusion}} Q_{\text{fusion}}$, it follows that $P_{\text{fusion}} \propto \underline{c}^4 \Gamma_{\text{nuc}}$, where Γ_{nuc} is the reaction rate of a highly localized nuclear reaction at the single site R_{fusion} .

To evaluate $\underline{c}^{\text{lower}}$ we determine an upper bound for Γ_{nuc} using a simplified reaction model for Eq. (5). This simplified model provides a higher reaction rate than is expected. We replace the eight nucleon problem of the reaction by an independent particle nucleon model⁴ in which the fusion process is modelled through a square well perturbation of the nucleonic potential. This square well is defined by a suitable upward shift in the kinetic energy T_0 of each D^+ ion over a characteristic nuclear dimension within the vicinity of R_{fusion} . This is equivalent to requiring in the reverse of reaction 5, an immediate dissipation of the energy of both alpha particles (which may be viewed as a short-lived virtual state) in the immediate region in which the nuclear reaction takes place. The height of the square well is $Q_{\text{fusion}} = 47.6$ MeV (calculated from the decrease in rest mass⁵), which is inferred from the assumption that the wave function overlap associated with the quadruple deuteron "multiple particle like object" may be viewed as an excited state of a ${}^8\text{Be}^{++++}$ nucleus.

Explicitly, using a suitable set of minimal uncertainty gaussian wave packets in which the initial state is defined with a characteristic electrostatic dimension $r_{z.p.} = 0.26$ Å and volume $V_{z.p.} = 4/3 \pi r_{z.p.}^3$, while the final state is confined to the nuclear volume, $V_{\text{nuc}} = 4/3 \pi R_{\text{Be}}^3$, defined by the Be^{++++} nuclear radius $R_{\text{Be}} = 9.108 \times 10^{-13}$ cm, we find that

$$\Gamma_{\text{fusion}} = 2\pi/\hbar [2 M_D R_{\text{Be}}^2 / (3\hbar^2)] (16/3)^{3/2} [V_{\text{nuc}}/V_{z.p.}] Q_{\text{fusion}}^2 \times (\underline{c}^{\text{lower}})^4, \quad (9)$$

where M_D is the mass of a deuteron and $\hbar = h/2\pi$. Using $P_{\text{fusion}} = 10 \text{ W/cm}^3 = N_{\text{site}} Q_{\text{fusion}} \Gamma_{\text{fusion}}$, where $N_{\text{site}} (\approx 4.81 \times 10^{22} \text{ cm}^{-3})$ is the density of Pd, and Eq. (9), we find that $\underline{c}^{\text{lower}}$ equals 2.8×10^{-7} . Though Eq. (9) was derived for an isolated perturbation, which applies to low temperature, the identical argument can be generalized to the case of higher temperature by performing a suitable averaging over initial state band energies in the Fermi Golden rule. In this case, it is still true that $P_{\text{fusion}} \propto \underline{c}^4 \Gamma_{\text{nuc}}$. However, Γ_{nuc} and \underline{c}^4 may both become strongly dependent on temperature due to disruption of Bloch symmetry by the fusion process. This feature could prove valuable for controlling the process.

MANAGING FUSION THROUGH THE BBC

The BBC D^+ fusion picture requires a suitable host lattice. One critical feature enabling condensed matter fusion is the establishment of conditions in which some D resides in the lattice in ionic form screened by itinerant electrons. A second requirement is that energies of the reactant single particle band be close to that of chemically bonded D so that it is possible to populate BBC states at realizable temperature. A third requirement is that the lattice provide adequate regularity and size to maintain approximate Bloch symmetry. A large $r_{z.p.}$ appears favorable and reflects a low interstitial D^+ binding energy, important for reducing electrostatic self-interaction. A large $r_{z.p.}$ favors high diffusivity. It is interesting that Pd provides a diffusivity⁶ for D that is two times greater than for H, contrary to expectation, suggesting that the bosonic character D^+ may contribute substantially to the diffusivity. This line of reasoning suggests that a Pd-Ag host, which provides enhanced H-diffusion, could be favorable for fusion reactions.

The key observable of BBC fusion is intense Alpha emission. For the 4-deuteron reaction (Eq.(5)), the Alpha particles have a range in Ilford emulsion of more than 240 microns⁵. Strong M- and N- shell x-ray line emission at -0.68 and -0.094 keV should be present. Neutrons are not predicted except as a result of Alpha bombardment of host impurities or of the surrounding electrolysis medium. Ionic superconductivity at sufficiently low temperature is expected.

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APPENDIX

DERIVATION OF EQUATION (9)

In this appendix, we provide additional detail concerning the derivation of Eq. (9). We evaluate Γ_{fusion} from the Fermi Golden rule:

$$\Gamma_{\text{fusion}} = 2\pi/\hbar \sum_f \delta(E_f - E_i) |\langle \Psi_f | \Delta V | \Psi_i^{\text{Bloch}} \rangle|^2 . \quad (\text{A-1})$$

Here, the many body Bloch bosonic initial state Ψ_i^{Bloch} involves a suitably symmetrized sum of products of single particle wave functions. Each product consists of a macroscopic number, N_b , of occupied single particle Bloch wave functions associated with a particular distribution of N_b distinguishable D^+ ions, in which not more than one single particle bosonic Bloch wave function ψ_i^{Bloch} is centered at a specific site, and each function has an associated energy band eigenvalue, ϵ_i , from the set $\{\epsilon_i, i=1, N_b\}$ of occupied bands. The sum over products is used to impose Bose symmetry, so that Ψ_i^{Bloch} is invariant when the coordinates of any number of its single particle wave functions are interchanged. Since each ion is located at one of N_b different sites, $\underline{c} = N_b/N_{\text{latt}}$, where N_{latt} is the total number of sites in the crystal. For the relevant case in which \underline{c} is small, each D^+ wave function ψ_i^{Bloch} is centered at a lattice site far from the other lattice sites.

The many body final state wave function Ψ_f of Eq. (A-1) consists of a similar sum of products of single particle wave functions, but for 4 fewer single particle Bloch states. In this case, N_b-4 band energies and wave vectors and the associated wave functions are selected, once, and for all, to be identical to N_b-4 eigenvalues from the initial state. Each term in the final state sum consists of a product of the associated N_b-4 initial state single particle wave functions, centered at different sites, and 4 single "defect" D^+ wave functions, which are all centered at the site R_{fusion} . Also in Eq. (A-1), \hbar is $h/(2\pi)$, ΔV is the self-interaction perturbation of the periodic potential.

Because of the important lattice-induced broadening of the charge and reduction in the electrostatic self-interaction energies $\Delta E^{(n)}$ for the virtual occupation by an n-particle Wannier state fluctuation, we may approximate ΔV by ΔV_{strong} . At low temperature (T), only isolated nucleonic perturbations need be considered because adjacent or near-by perturbations result in increases in entropy and free energy of the crystal system. Near T=0, there is only a single site (R_{fusion}) where ΔV_{strong} is non-vanishing. Then, in the evaluation of the transition matrix element $\langle \Psi_f | \Delta V_{\text{strong}} | \Psi_i^{\text{Bloch}} \rangle$, because ΔV_{strong} is short-ranged and each single particle initial state wave function possesses Bloch symmetry, two important consequences follow from Eq. (1). 1) Large contributions to the perturbation matrix element result from the product consisting of any four single particle Bloch states throughout the lattice in the initial state multiplied by the highly localized perturbation with the four defect final state ψ_f functions. 2) All contributions to the matrix element are negligible except for those associated with the integrations over regions near the site where interaction takes place.

The number of distinct contributions that arise from the overlap of each of the Bloch states with the 4 defect states in the region near R_{fusion} is $N_b(N_b-1)(N_b-2)(N_b-3) = N_b^4$, for $N_b \gg 1$. Then, when each single particle Bloch state is normalized to the volume of the crystal, it follows as a consequence of Eqs. (1) and (A-1) that

$$\Gamma_{\text{fusion}} = 2\pi / \hbar \left[\sum_f \left(\left| \Delta V_{f,j}^{\text{strong}} \right|^2 \rho_f(e_j) \right) \right] \Omega^4 . \quad (\text{A-2})$$

Here $\Delta V_{f,j}^{\text{strong}}$ is the matrix element derived from integration over the region of volume V_{nuc} , centered at R_{fusion} , where the product of ΔV_{strong} with the four localized final state single particle wave functions and four initial state functions is non-negligible. Here, $\rho_f(e_j)$ is the final density of states evaluated with energy $e_f = \sum_j e_j$, where the summation includes the four initial state bands that are excluded from the final state.

As discussed in the text, to establish $\underline{c}^{\text{lower}}$ in Eq. (9), it is sufficient to consider a faster, simpler reaction based on Eq. (5): we have replaced the

eight nucleon problem of the reaction by an independent particle nucleon model⁴ in which the fusion process is modelled through a square well perturbation of the nucleonic potential. This square well is defined by a suitable upward shift in the kinetic energy T_0 of each D^+ ion over a characteristic nuclear dimension within the vicinity of R_{fusion} . This is equivalent to requiring immediate dissipation of the energy of both alpha particles (which may be viewed as a short-lived virtual state) in the reversed reaction in the immediate region in which the nuclear reaction takes place.

Then, each defect and initial state function eigenvalue is measured relative to the constant energy scale associated with a perfectly ordered environment in which no fusion occurs. The height of the square well is $Q_{\text{fusion}} = 47.6$ MeV, which is inferred from the assumption that the wave function overlap associated with the quadruple deuteron "multiple particle like object" may be viewed as an excited state of a ${}^8\text{Be}^{++++}$ nucleus and from the associated decrease in rest mass due to the decay of the unstable ${}^8\text{Be}^{++++}$ nucleus to two alpha particles.

Explicitly, we find

$$\Gamma_{\text{fusion}} = 2\pi/\hbar (|O(J)|^2 \rho_{\pm}(e_J)) \leq^4 Q_{\text{fusion}}^2, \quad (\text{A-3})$$

where the overlap matrix element $O(J)$ is given by

$$O(J) = \prod_{f' \text{ and } j = 1,4} \int_{\text{nuc}} \psi_{f'}^*(r') \psi_j^{\text{Bloch}}(r') d^3r', \quad (\text{A-4})$$

and each integral in Eq. (A-4) extends only over the region near R_{fusion} where the values of ΔV_{strong} associated with the square well are non-negligible. Because $\Delta V_{\text{elec}} \ll \Delta V_{\text{strong}}$, we may rewrite each product of four initial and final state wave functions in a separable, Born-Oppenheimer form, involving a slowly varying envelope function ψ_{CM} whose spatial dependence only involves the center of mass coordinate R_{cm} ($\psi_{\text{CM}} = \psi_{\text{CM}}(R_{\text{cm}})$), multiplied by three more rapidly varying functions, ψ^{nuc} , whose coordinate dependencies are all expressed relative to R_{cm} . Then, we obtain a reasonable upper bound for $|O(J)|^2 \rho_{\pm}(e_J)$ by considering appropriate bounds for the single integral over the center of mass coordinate of the product $\psi_{\text{CM},\pm} \psi_{\text{CM},i}$ and including only the dependence of the

associated change in total energy in $\rho_f(e_J)$ using the single particle density of states that is appropriate for $\psi_{CM,f}$. Thus, we find

$$O(J) = \int_{nuc} \psi_{CM,f}^*(r') \psi_{CM,i}^{Bloch}(r') d^3r' , \quad (A-5)$$

where the single integration extends over the volume V_{nuc} centered at R_{fusion} because ΔV_{strong} is non-vanishing only in this region. Then, the maximum in $O(J)$ occurs when $\psi_{CM,f}$ is primarily localized within V_{nuc} but has considerable overlap with $\psi_{CM,i}$. This occurs when $\psi_{CM,f}$ is appreciable only on length scales associated with nuclear interaction while $\psi_{CM,i}$ has variation on a considerably larger length scale.

We derive \underline{c}^{lower} from Eqs. (A-3)-(A-5) using initial and final states, approximated from minimal uncertainty wave-packets, $\psi_{packet,i}$ and $\psi_{packet,f}$ that incorporate this interplay between length scale. Each packet possesses a suitable characteristic variance ($\langle r_{characteristic}^2 \rangle$) in position associated with the minimal error in the measurement of position and momentum. Specifically,

$$\psi_{packet,i} = N_o \exp(-r^2/r_{z.p.}^2) , \quad (A-6)$$

where $N_o = (2/(\pi r_{z.p.}^2))^{3/4}$, and

$$\psi_{packet,f} = N_f \exp(-3r^2/(4 R_{Be}^2)) , \quad (A-7)$$

and $N_f = (3/(2\pi R_{Be}^2))^{3/4}$. Here, $R_{Be} = 9.108 \times 10^{-13}$ cm, as derived from a second calculation associated with the independent nucleon model of Be in which all of the nucleons are bound by a square well of depth $V_o = 50$ MeV, the number of bound nucleon states is precisely 9, and the associated Fermi energy for the eight nucleon case (~ 17 MeV below the top of the well) is typical for nuclear matter⁴. Also, $V_{nuc} = 4/3\pi R_{Be}^3$. Since in Eq. (A-6), realistic values of $r_{z.p.}$ can be as large as a significant percentage of the nearest neighbor Pd separation A_{Pd} (2.75 A), we have used $r_{z.p.} = 0.5 B = 0.265 \text{ \AA} \approx 0.1 A_{Pd}$. Then, $V_{nuc}/V_{z.p.} \sim 10^{-12}$, where $V_{z.p.} = 4/3 \pi r_{z.p.}^3$ is the total volume of each D^+ due to zero point motion. From Eqs. (A-5)-(A-7), we find

$$O(J) = (16/3)^{3/4} (V_{nuc} / V_{z.p.})^{1/2}, \quad (A-8)$$

where we have approximated $4R_{Be}^2 + 3r_{z.p.}^2$ with $3r_{z.p.}^2$ since $R_{Be}^2 \ll r_{z.p.}^2$. The appropriate final density of states is

$$\rho_f(0) = 1/(\hbar \omega_{nuc}), \quad (A-9)$$

where $\omega_{nuc} = 3/2 \hbar / (M_D R_{Be}^2)$. Equation (A-9) follows from Eq. (A-7) because $\psi_{packet,f}$ is the ground state of a three dimensional harmonic oscillator that possesses an angular frequency of ω_{nuc} . Substituting Eqs. (A-8) and (A-9) into Eq. (A-1), we find

$$\Gamma_{fusion} = 2/3 \pi (16/3)^{3/2} (Q_{fusion}^2 (\kappa R_{Be})^2 \times V_{nuc}) / (2 \hbar V_{z.p.} V_o) (\underline{c}^{lower})^4, \quad (A-9)$$

where $\kappa = (V_o M_D 2/\hbar^2)^{1/2}$, which is equivalent to Eq. (9).