EXAMINATION OF A PROPOSED PHONON-COUPLING MECHANISM FOR COLD FUSION

OAKLEY H. CRAWFORD *Oak Ridge National Laboratory*
Health and Safety Research Division, Oak Ridge, Tennessee 37831-6123

COLD FUSION

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The proposed nuclear energy in an atomic lattice (NEAL) mechanism for nuclear fusion in a cathode during electrolysis of D_2O is examined. In this mechanism, coupled harmonic motion of deuterons is supposed to lead to a reduction in the width of the Coulomb barrier for proton-deuteron (p-d) fusion in palladium, thereby substantially increasing the fusion rate. Instead, it is argued that deuteron-deutron coupling does not have an important effect and that interaction with phonons does not enhance the p-d fusion rate.

Schwinger[1,2] proposed the nuclear energy in an atomic lattice (NEAL) mechanism by which nuclear fusion might occur in a palladium cathode during electrolysis of D_2O. The process would be cold fusion as claimed by Fleischmann et al.,[3] except that the dominant reaction would be $p + d \to {}^3He$ + heat. In this mechanism, coupled harmonic motion of the deuterons is supposed to lead to a reduction in the width of the Coulomb barrier for proton-deuteron (*p-d*) fusion in palladium, thereby substantially increasing the fusion rate. However, it is concluded below that these deuteron motions do not appear to reduce significantly the barrier width or to introduce a new effect by which the *p-d* fusion rate might be enhanced. Besides the application to cold fusion, this work is relevant to the study of fundamental physical interactions of charged particles with solids.

In the NEAL mechanism,[2] stationary palladium atoms define a regular array of lattice points fixed in space about which N deuterium positive ions (deuterons) oscillate in coupled harmonic motion. Anisotropy of phonon modes is ignored. A proton entering the system is assumed, for simplicity, to interact with only one particular deuteron, to be called number 1 (and not with the metal atoms). The Hamiltonian H for the system is

$$H = H_L + T_p + V \ , \tag{1}$$

where

H_L = Hamiltonian for motion of the deuterons, i.e., for phonons

T_p = kinetic energy of the proton

V = *p-d* interaction potential, which by assumption depends only on the distance R_{pd} between the proton and deuteron 1.

The results are specialized to the case of a bare Coulomb repulsion, i.e., $V = e^2/R_{pd}$, where e is the elementary charge.

The wave function of this system can be expanded over the stationary states ϕ of coupled motion of the deuterons as follows:

$$\Psi = \sum_{\phi} |\phi\rangle \psi_\phi(\boldsymbol{R}_p) \ . \tag{2}$$

Assuming the deuteron system is initially in the ground state (no phonons), denoted by $\phi = 0$, elastic scattering of the proton is described by the function $\psi_0(\boldsymbol{R}_p)$. The latter can in principle be computed from a one-particle Schrödinger equation,[4] in which the effective potential $\mathcal{V}$ (called the optical potential) is in general a complex, energy-dependent function of $\boldsymbol{R}_p$. Schwinger derived both exact and approximate expressions for $\mathcal{V}$. In particular, he found[2] an asymptotic expression, valid when the distance R_p of the proton from the specific lattice point is large. It can be written as follows:

$$\mathcal{V} \sim \frac{e^2}{R_p} - \frac{e^4}{2MR_p^4}\left\langle \frac{1}{\omega^2} \right\rangle \ , \tag{3}$$

where

$\langle 1/\omega^2 \rangle$ = average of the inverse square of the phonon angular frequency over all $3N$ modes

M = deuteron mass.

(Recall that the phonons are those of the system of deuterons; the palladium atoms are stationary by assumption.)

The first term in the foregoing asymptotic expression is just the average of $V = e^2/R_{pd}$ over the ground state, or phonon vacuum. The second term arises from the response of the deuterons to the proton, i.e., to the creation of virtual phonons. Since this term is negative, it was inferred that interaction with phonons[a] narrows the barrier for *p-d* fusion.[2] However, $\mathcal{V}$ is the effective potential for the motion of the proton relative to the fixed lattice, *not relative to a deuteron*. Therefore, $\mathcal{V}$ does not directly relate to the barrier for fusion. Still, it is worthwhile to determine the source of the foregoing negative term in the asymptotic $\mathcal{V}$.

[a]To include lattice coupling or interaction with phonons simply means to allow the deuterons (particularly number 1) to adjust to the presence of the proton, which necessarily implies an admixture of phonons. Note that deuteron-deuteron (*d-d*) coupling is not an essential feature; in its absence, interaction with phonons implies that deuteron 1 does not have precisely a ground-state harmonic oscillator wave function.

The polarizability α of the model system of deuterons, in which only deuteron 1 is affected by the field, is given by the following sum over phonon modes:

$$\alpha = 2e^2 \sum_q \frac{\langle 0|x_1|q\rangle\langle q|x_1|0\rangle}{\hbar\omega_q} \ , \tag{4}$$

where x_1 is the displacement, in the direction of the external field, of deuteron 1. Only single-phonon states contribute to the foregoing sum because of the form of the x_1 operator,

$$x_1 = \sum_q \rho_q (a_{qx} + a_{qx}^\dagger) \ , \tag{5}$$

where a_{qx} and $a_{qx}^\dagger$ are phonon destruction and creation operators $[a_{qx}^\dagger, a_{px}] = \delta_p^q$ and $\rho_q = (\hbar/2M\omega_q N)^{1/2}$. The result is

$$\alpha = \frac{e^2}{M}\left\langle \frac{1}{\omega^2} \right\rangle \ . \tag{6}$$

Therefore, the original asymptotic expression (3) for $\mathcal{V}$ is equivalent to the following:

$$\mathcal{V} \sim \frac{e^2}{R_p} - \frac{\alpha e^2}{2R_p^4} \ . \tag{7}$$

(Note the resemblance to electron scattering from a spherically symmetric ion, for which case the optical potential goes asymptotically[4] to $-Ze^2/r - \alpha_a e^2/2r^4$, where Ze is the charge of the ion, r is the distance of the electron from the nucleus, and α_a is the ion dipole polarizability.)

This last result suggests that the negative term in the asymptotic form of $\mathcal{V}$ arises from polarization of the deuteron system. It seems clear that this is due to motion of the particular deuteron away from the proton, but let us verify this. Apply the adiabatic approximation, which is valid for $R_p \gg v/\langle\omega\rangle_{Av}$, where v is the velocity of the proton and $\langle\omega\rangle_{Av}$ is an appropriate average phonon frequency.[4] Then, the target responds to the proton as though the latter were at rest. Consequently, for any specified (large) R_p, the wave function for the deuteron system is an eigenfunction of $H_L + V$, and $\mathcal{V}$ is the corresponding eigenvalue. Expansions of this eigenfunction and eigenvalue in powers of R_p^{-1} can readily be derived to low orders in the standard way from perturbation theory, approximating the perturbation $V = e^2/R_{pd}$ by $e^2/R_p - e^2x_1/R_p^2$, where x_1 is given by Eq. (5). The resulting expansion of the eigenvalue through fourth order in R_p^{-1} is identical to Eq. (7) for $\mathcal{V}$, showing that the adiabatic approximation is consistent with the asymptotic development in Ref. 2. More to the point, from the resulting wave function, one finds that the perturbation increases the mean value of R_{pd} by the amount of α/R_p^2 to lowest order. Thus, the lowering of the optical potential at large R_{pd} is directly associated with motion of the particular deuteron away from the proton.

The implications of the foregoing results can be stated in terms of screening but not, however, by the other deuterons or by the electrons. (Screening by the former is ignored in this mechanism, and screening by the latter is not of central importance.) As outlined earlier, the optical potential $\mathcal{V}$ is reduced at long range when the particular deuteron is allowed to respond to the field of the proton. (Coupling to the other deuterons is of secondary import.) This reduction constitutes a screening of $\mathcal{V}$ (but not of V), by deuteron 1! The fact that it is accomplished by evasive motion of deuteron 1 implies a decrease in the *p-d* fusion rate. It seems clear, therefore, that properly treating deuteron motions would lead to smaller calculated *p-d* fusion rates than if phonons were neglected.

In summary, the optical potential $\mathcal{V}$ depends on R_p and is thus not directly related to the Coulomb barrier for fusion. Anyway, the negative term $-\alpha e^2/2R_p^4$ in the asymptotic expansion for $\mathcal{V}$ appears even in the absence of *d-d* coupling (although with different α)[b] and is furthermore associated with motion of the particular deuteron *away* from the proton, as shown above. In addition, it is clear that at short range (small R_{pd}), *d-d* interactions can justifiably be omitted from the Hamiltonian and that deuteron 1 still tends to move away from the proton. I conclude that the coupling between deuterons does not appear to have a large effect on the *p-d* fusion rate in a deuterated solid and that interaction with phonons, in the sense of the NEAL mechanism, does not enhance the *p-d* fusion rate.

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REFERENCES

1. J. SCHWINGER, "Cold Fusion: A Hypothesis," *Z. Naturforsch.*, **45a**, 756 (1990).

2. J. SCHWINGER, "Nuclear Energy in an Atomic Lattice. 1," *Z. Phys. D*, **15**, 221 (1990).

3. M. FLEISCHMANN and S. PONS, "Electrochemically Induced Nuclear Fusion of Deuterium," *J. Electroanal. Chem.*, **261**, 301 (1989); see also Errata, *J. Electroanal. Chem.*, **263**, 187 (1989); see also M. FLEISCHMANN, S. PONS, M. W. ANDERSON, L. J. LI, and M. HAWKINS, "Calorimetry of the Palladium-Deuterium-Heavy Water System," *J. Electroanal. Chem.*, **287**, 293 (1990).

4. M. H. MITTLEMAN and K. M. WATSON, "Scattering of Charged Particles by Neutral Atoms," *Phys. Rev.*, **113**, 198 (1959).

[b]In the absence of coupling between deuterons, $\langle 1/\omega^2\rangle$ is replaced by $1/\omega_0^2$ in Eq. (6) for the polarizability, where ω_0 is 2π times the frequency of oscillation of the deuteron about its equilibrium point.