

# ESTIMATE OF THE NEUTRON TRANSFER FUSION RATE

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The  $e^4$ -order quantum electrodynamic term for neutron transfer fusion is  $10^{40}$  to  $10^{50}$  times larger than the direct term, suggesting that room-temperature fusion does not contradict nonexotic physics.

In addition to many negative findings, an increasing number of laboratories are reporting observations of cold fusion.<sup>1-5</sup> As is well known, the observed fusion rate exceeds the theoretical estimates by factors of  $10^{25}$  to  $10^{35}$ . In a previous paper,<sup>6</sup> it was pointed out that when the lowest order term is suppressed, higher order terms may take over and be responsible for the reaction under consideration. This theory was applied to the cold fusion process where  $d$ - $p$  fusion was considered in order  $e^2$ , and it was shown that the probability of the process may be substantially enhanced over that given by the direct term. In this technical note, we consider the process of neutron transfer fusion to order  $e^4$  in the electromagnetic coupling; in particular, we work out the reaction amplitude according to Figs. 2d and 2e of Ref. 6.

It must be emphasized that the evaluation of the graphs can yield only a result that largely overestimates the reaction probability since we do not at this time incorporate any specific information about the stereochemistry of the actual reaction. Such molecular/solid-state effects usually result in large (four or five orders of magnitude) suppression factors of the reaction rates. Our calculation must be considered to represent only the skeleton of the actual reaction.

For concreteness, we assume the following process. We consider that a deuteron of momentum  $k_3$  impinges on the bound-state system of nuclei  $A$  and  $C$  with momenta  $k_1$  and  $k_2$ , respectively, the momentum space wave function of which we denote as  $\Psi(P)$  [ $\Psi(R_{AC})$  in position space]. The binding energy is assumed to be  $\sim 0.1$  eV, corresponding to solid-state or molecular dynamics. After the neutron transfer fusion reaction, we assume that the final-state nuclei  $C$ ,  $B$ , and  $T$  are free and have momenta  $k'_1$ ,  $k'_2$ , and  $k'_3$ , respectively. For nucleus  $A$ , we have in mind  $^2\text{H}$  or  $^7\text{Li}$ ; nucleus  $C$  could be palladium or oxygen, corresponding to electrolytically or chemically induced fusion, respectively. Of course, the treatment is the same for any neutron transfer fusion process.

COLD FUSION

TECHNICAL NOTE

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Since all participating nuclei are strictly nonrelativistic, the Feynman graph in Fig. 1 can be replaced by its nonrelativistic time-ordered form, i.e., by the appropriate reaction graph, Fig. 2. At the same time, only the Coulomb part of the photon propagator survives.

Before actually evaluating Fig. 2, we illustrate its meaning in terms of the usual position space Rayleigh-Schrödinger perturbation expansion. In the second-order matrix element

$$T_{fi} = \sum_m \frac{\langle \Psi_f | V_2 | \Psi_m \rangle \langle \Psi_m | V_1 | \Psi_i \rangle}{E_m - E}, \quad (1)$$

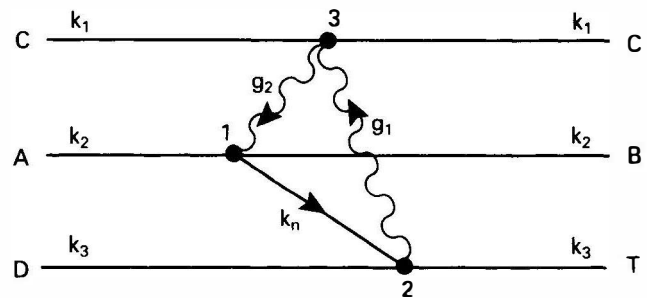


Fig. 1. Second-order quantum electrodynamic correction to transfer fusion.

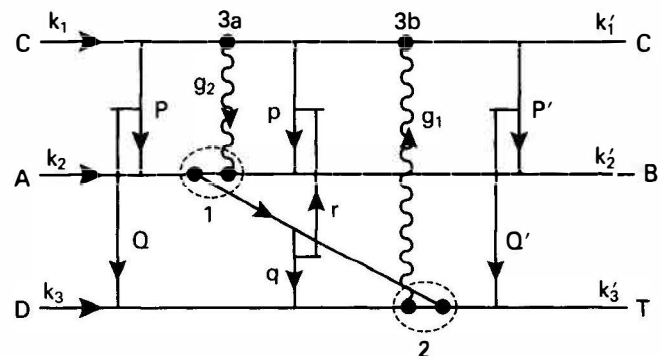


Fig. 2. Nonrelativistic model for Fig. 1: momentum space reaction graph.

the participating wave functions have the following meanings:

$$\Psi_i = \Psi(R_{AC})\psi_C(R_C, \rho_i)\psi_A(R_A, \rho_{Bn})\psi_D(R_D, \rho_k) \quad (2)$$

and

$$\Psi_f = \psi_C(R_C, \rho_i)\psi_B(R_B, \rho_i)\psi_T(R_T, R_D, \rho_{nD}) \quad (3)$$

while the intermediate state  $\Psi_m$  is

$$\Psi_m = \psi_C(R_C, \rho_i)\psi_B(R_B, \rho_i)\psi_D(R_D, \rho_k)\psi_n(r_n) \quad (4)$$

The interaction potentials are

$$V_1 = \frac{e^2 Z_C Z_D}{|R_C - R_D|} \quad (5)$$

and

$$V_2 = \frac{e^2 Z_C Z_B}{|R_C - R_B|} \quad (6)$$

In these equations, the  $R_i$  are center of mass (CM) coordinates, while the  $\rho_i$  are relative coordinates. The energy denominator in Eq. (1) begins at the  $(\gamma, n)$  threshold of nucleus  $A$ , which is 2.2 MeV for the deuteron and 0.6 MeV for  ${}^7\text{Li}$ . We evaluate this matrix element in terms of the reaction graph in momentum space.

The vertices of Fig. 1 have the following meanings. Vertex 1 describes the photodisintegration of nucleus  $A$ , yielding nucleus  $B$  and a neutron. Vertex 2 is that of the (inverse) photodisintegration of the triton. Vertex 3 is that of the (elastic) Compton scattering off nucleus  $C$ . Since all of these vertices concern off-the-mass-shell processes, we replace them by the simplest possible models, as indicated in Fig. 2. Vertex 1 is given by the appropriate spectroscopic factor  $S_A(\rho_2)$  of the  $n$ - $B$  system (depending on nucleus  $A$ , order of magnitude  $10^{-1}$  to  $10^{-2}$ ), while the (Coulomb) photon interacts with the (charged) recoiling nucleus  $B$ . Similarly, vertex 2 is described by the spectroscopic factor  $S_T(\rho_3)$  of the  $D$ - $n$  system of tritium (order of magnitude unity), while the photon interacts with the recoiling nucleus  $D$ . Finally, the (Coulomb) photon scattering vertex 3 is described by the two photon/ $C$  interaction vertices and the propagator carrying the system from vertex 3a to 3b.

We employ the momentum space Jacobi coordinates also indicated on Fig. 2: The initial and final-state three-body systems are given in terms of the Jacobi coordinates  $P$  and  $Q$ , and  $P'$  and  $Q'$ , respectively, while the intermediate four-body system is described by the Jacobi coordinates  $p$ ,  $q$ , and  $r$ . The overall CM coordinate is written as  $K$ . All these coordinates are three-vectors. We thus have the following kinematic relations:

$$g_1 = (k'_2 - k_2) + (k'_1 - k_1) + k_n \quad (7)$$

$$g_2 = k'_2 - k_2 + k_n \quad (8)$$

$$p = [m_c k'_2 - m_B(k - k'_2 + k_2 - k_n)]/M_{CB} \quad (9)$$

$$q = (m_n k_3 - m_D k_n)/M_{nD} \quad (10)$$

$$r = [M_{Dn}(k_1 + k_2 - k_n) - M_{CB}(k_n + k_3)]/M_{BCDn} \quad (11)$$

$$P = (m_C k_2 - m_A k_1)/M_{AC} \quad (12)$$

$$Q = [M_{AC} k_3 - m_D(k_1 + k_2)]/M_{ACD} \quad (13)$$

$$K = k_1 + k_2 + k_3 = K' = k'_1 + k'_2 + k'_3 \quad (14)$$

$$P' = (m_C k'_2 - m_B k'_1)/M_{BC} \quad (15)$$

and

$$Q' = [M_{BC} k'_3 - m_T(k'_1 + k'_2)]/M_{BCT} \quad (16)$$

In these equations, the  $m_i$  are the masses of nuclei  $i$ ;  $M_{ij} = m_i + m_j$ ,  $M_{ijk} = m_i + m_j + m_k$ , etc.; and the momenta are those shown in Fig. 2.

Considering nuclei  $A$  and  $T$  to have  $s$ -wave neutron channels,  $S_A(0) \neq 0$  and  $S_T(0) \neq 0$ . Since nuclei  $A$  and  $T$  contain reasonably large (nuclear) momenta, we neglect their momentum dependence; i.e., we replace  $S_A(\rho_1)$  with  $S_A(0)$  and  $S_T(\rho_2)$  with  $S_T(0)$  and extract them from the integral. (This leads to a minor overestimate of the probability.)

We now can write down the matrix element corresponding to the graph. We have

$$M(P, Q, K; P', Q', K) = e^4 Z_C Z_B S_A(0) Z_C Z_D S_T(0) I(P, Q, K; P', Q', K) \quad (17)$$

and

$$I(P, Q, K; P', Q', K) = \int \frac{d^3 k_n}{(2\pi)^3} \frac{1}{g_2^2} \frac{1}{\frac{p^2}{2\mu_1} + \frac{q^2}{2\mu_2} + \frac{r^2}{2\mu_3} + (E + \epsilon_B)} \frac{1}{g_1^2} \quad (18)$$

where

$$\mu_1 = \frac{m_B m_C}{M_{BC}} \quad ,$$

$$\mu_2 = \frac{m_D m_n}{M_{Dn}} \quad ,$$

and

$$\mu_3 = \frac{M_{BC} M_{Dn}}{M_{BCDn}} \quad . \quad (19)$$

Using Eqs. (7) through (16), we rewrite Eq. (18) as

$$I(P, Q, K; P', Q', K) = \Lambda \int \frac{d^3 k_n}{(2\pi)^3} \frac{1}{(A - k_n)^2 (B - k_n)^2 [(C - k_n)^2 + D^2]} \quad (20)$$

where

$$A = k_1 - k'_1 + k_2 - k'_2 \quad , \quad (21)$$

$$B = k_2 - k'_2 \quad , \quad (22)$$

$$C = -\frac{f_1}{2f_2} \quad , \quad (23)$$

$$D^2 = \frac{f_0}{f_2} - \left(\frac{f_1}{2f_2}\right)^2 \quad , \quad (24)$$

and

$$\Lambda = \frac{1}{f_2} \quad , \quad (25)$$

where

$$f_2 = \frac{M_{BC}}{m_B m_C} + \frac{m_D}{M_{Dn} m_n} + \frac{M_{BC}}{M_{Dn} M_{BCDn}} \quad , \quad (26)$$

$$f_1 = -\frac{k_2}{\mu_1} + \frac{k'_2}{\mu_2} - \frac{k_3}{M_{Dn}} \quad , \quad (27)$$

and

$$f_0 = \frac{1}{2} \left( k'_1 + k'_2 - \frac{M_{BC}}{m_B} k_2 \right)^2 \frac{m_B}{M_{BC} m_C} + \frac{1}{2} k_3^2 \frac{m_D}{M_{Dn} m_n} + \frac{1}{2} \left[ k'_1 + k'_2 - \frac{M_{BC}}{M_{Dn}} k_3 \right]^2 \frac{M_{Dn}}{M_{BC} M_{BCDn}} - (E + \epsilon_B) . \quad (28)$$

Note that Eq. (18), and hence also Eq. (20), has no singularities for real  $k_n$ . This can be seen as follows. Breaking the domain of integration of Eq. (20) into three regions, i.e., around  $k_n \approx A$ , around  $k_n \approx B$ , and the rest, one sees by shifting the integration variable to  $x = k_n - A$  and  $x = k_n - B$  for the first two regions, respectively, that in fact no singularity remains. Therefore, the integrals given below have to be taken as principal value integrals.

The integral (20) now can be transformed in the usual manner by defining the shift in the integration variable

$$x = k_n - C \quad (29)$$

and introducing the auxiliary integration to yield

$$I(P, Q, K; p', Q', K) = \frac{6\Lambda}{(2\pi)^3} \int_0^1 d\alpha (1 - \alpha) \alpha J(\alpha) , \quad (30)$$

where

$$J(\alpha) = \int d^3x \frac{1}{[R(\alpha) - x]^4 (x^2 + D^2)} , \quad (31)$$

and where

$$R(\alpha) = \alpha A + (1 - \alpha) B - C . \quad (32)$$

The integral (30) now can be worked out in terms of elementary functions. To obtain the transition amplitude, we now have to account for the initial bound state. Thus,

$$U = \int d^3P \Psi(P) M(P, Q, K; P', Q', K) . \quad (33)$$

We now recognize that  $\Psi(P)$  has support only for  $|P|$  of molecular or solid-state scale, while the main contributions to  $I$  involve nuclear-scale momenta. Throughout, e.g., Eqs. (7) and (8), one can thus neglect the unprimed (i.e., initial-state) variables with respect to the primed variables. We are thus led to

$$U = M(0,0,0; P'Q'0) \int d^3P \Psi(P) . \quad (34)$$

Assuming now for the bound-state wave function

$$\Psi(P) = \left( \frac{8\kappa_L^5}{15\pi^2} \right)^{1/2} \frac{1}{(P^2 + \kappa_L^2)^2} , \quad (35)$$

we find

$$U = \left( \frac{8\pi^2 \kappa_L^3}{15} \right)^{1/2} M(0,0,0; P'Q'0) . \quad (36)$$

This concludes the formal developments.

Since this treatment does not include molecular/solid-state dynamics, a precise evaluation of the cross section at

this stage is not worthwhile. We therefore give an order-of-magnitude evaluation of the reaction rate

$$W = \sigma \rho v , \quad (37)$$

where  $\rho \approx 10^{22} \text{ cm}^{-3}$  is the density of  $(AC)$  bound systems in the target,  $v \approx 10^5 \text{ cm/s}$  is the velocity of the incoming deuterons, and  $\sigma$  is the cross section:

$$\sigma \approx |U|^2 \rho_f(E) . \quad (38)$$

The final state three-body phase-space is

$$\rho_f(E) \sim P_f^5 , \quad (39)$$

where  $P_f$  is a characteristic momentum of the final three-body state:

$$P_f \sim (2\mu_f E_f)^{1/2} , \quad (40)$$

where  $E_f$  is the reaction  $Q$  value and  $\mu_f$  is the appropriate reduced mass. In our case,  $E_f \approx 5 \text{ MeV}$ ,  $\mu_f \approx 2000 \text{ MeV}$ , and  $P_f \approx 140 \text{ MeV}$  ( $\hbar = c = 1$ ).

We now turn to the matrix element itself. To estimate its magnitude, we consider the form of Eq. (18). Because of its strong convergence and because no resonances occur, the main contributions to the integral result from small momenta. We *substantially underestimate* its magnitude by replacing it with

$$I \rightarrow \frac{k_n^3}{(2\pi)^3} \frac{1}{k_n^2} \frac{1}{4E} \frac{1}{k_n^2}$$

when taking  $k_n \approx 200$  and  $E = 5$ . We have  $I \approx 2 \times 10^{-6}$ . Finally, we specify  $C = {}^{16}\text{O}$ ,  $A = {}^7\text{Li}$ , and  $D = {}^2\text{H}$  to obtain the estimate for the rate:

$$W_{th} \approx [S_A(0)S_T(0)]^2 \times 10^{-5} \text{ s}^{-1} ,$$

which should be compared with the experimental rate of neutron production ( $W_n \approx 10^{-22} \text{ s}^{-1}$ ) and of heat production ( $W_h \approx 10^{-12} \text{ s}^{-1}$ ), as quoted elsewhere.

At this point, it is worthwhile to briefly discuss some aspects of the influence of the surroundings and the way they might modify the computed fusion rate. A prominent consideration is provided by analogy with other physical processes associated with the Coulomb interaction, e.g., bremsstrahlung or pair production, which, as is well known, are decisively influenced by screening of the nuclear charge by the surrounding electrons. This effect plays no role in the present context for two reasons. First, the reaction amplitude, being analogous to Compton scattering, is quadratic in the charge of particle  $C$ . Thus, rather than yielding a cancellation between the contributions from the nuclei and from the electrons, as is the case in low-momentum transfer bremsstrahlung (i.e., the screening effect), here the amplitudes add.

Furthermore, in the present case, the typical momentum transfers are some hundreds  $\text{fm}^{-1}$ . The process is therefore inelastic in that particle  $C$  is knocked out of its initial bound state, thus eliminating the possibility of interference, either destructive or constructive. Hence, the contributions from the different neighboring catalyzing particles, both nuclei and electrons, simply add as probabilities. Thus, as far as the basic reaction mechanism is concerned, the surroundings only increase the reaction rate. The investigation of the actual inhibition factor, evident in the ratio of observed to computed fusion rate, is far beyond the scope of this technical

note. However, our result, which is 40 to 50 orders of magnitude larger than the conventional results, seems to indicate that the existence of room-temperature fusion under favorable conditions is compatible with the present-day understanding of physics and requires no special mechanism for its explanation.

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