THE FUSION RATE IN THE TRANSMISSION RESONANCE MODEL

COLD FUSION

TECHNICAL NOTE

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Resonant transmission of deuterons through a chain of target deuterons in a metal matrix has been suggested as an explanation for the cold fusion phenomena. The fusion rate in such transmission resonance models is estimated, and the basic physical constraints are discussed. The dominating contribution to the fusion yield is found to come from metastable states. The fusion rate is well described by the Wentzel-Kramer-Brillouin approximation and appears to be much too small to explain the experimental anomalies.

INTRODUCTION

A stream of particles incident on a pair of high and wide potential barriers can, for certain discrete energies, be perfectly transmitted, although a single barrier would almost completely reflect the incident particles. This transmission resonance effect is a well-known consequence of the wave nature of matter in quantum mechanics.[1]

Turner[2,3] suggested that the transmission resonance effect could be the key to understanding cold fusion in metals. Bush[4] has constructed an extensive phenomenological model, based on the resonant transmission of mobile deuterons through a chain of Coulomb barriers provided by fixed deuterons in a metal lattice. His model was found to agree with a wide range of reported cold fusion phenomena.[4] Bush's model focuses on the deuteron current,

$$j = \frac{ih}{2m}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi) ,$$

which, at resonance, smoothly flows through the model system. Explicit estimates of the fusion rate are, however, not given.

The fusion rate of a pair of deuterons is conventionally taken to be $K|\Psi(r_0)|^2$, where $\Psi(r_0)$ is the wave function of the deuteron system at the nuclear radius r_0, and $K = 1.5 \times 10^{-16}\ \mathrm{cm}^3\cdot\mathrm{s}^{-1}$ is a constant that includes the nuclear matrix element. The wave function is for sufficiently low energies well estimated in the Wentzel-Kramer-Brillouin (WKB) approximation.

Deuterons embedded in dense matter interact with a screened potential $V(r)$. The classical outer turning point b for a deuteron pair of energy E is defined by $E = V(b)$. It is convenient to simplify the problem by assuming that the deuteron density, in the classically allowed region, is of the same order of magnitude as the average deuteron density ρ. Using the connection formulas of the WKB method, we estimate the fusion rate per deuteron to be

$$\lambda = \rho K \exp\left(-\frac{2}{\hbar}\int_0^b \{2\mu[V(r) - E]\}^{1/2}\right) , \qquad (1)$$

where μ is the reduced mass of the deuteron pair. If one of the deuterons is considered to be a fixed target, μ is simply the deuteron mass.

The fusion yield predicted by this expression is, for any realistic choice of screened potential, much smaller than the anomalous cold fusion rates that have been reported by several experimental groups.[5,6] Collective effects involving the coherent dynamics of many deuterons could possibly enhance the cold fusion yield beyond the baseline level given by Eq. (1). The suggested transmission resonance effect is a simple example of such effects.

In the following, we show, however, that the fusion rate in transmission resonance models is well described by the WKB approximation [Eq. (1)]. This conclusion follows from the generic properties of transmission resonances and appears to be stable against any variations of, e.g., the barrier configuration or the preparation of the incident wave packet.

THE METASTABLE STATE GENERATES THE FUSION YIELD

Bohm[1] has given a detailed and pedagogic description of transmission resonances. We collect, simplify, and discuss some of the results that are of particular importance for cold fusion models.

Consider a pair of identical potential barriers as shown in Fig. 1. Each potential barrier is symmetric with respect to reflection in the highest point. We assume that $V = 0$ away from the barriers (regions I, III, and V). The key parameter of the barriers is the barrier penetration factor,

$$\theta = \exp\left(\int_{-b}^{b} \frac{1}{\hbar}\{2\mu[V(r) - E]\}^{1/2}\right)$$
$$= \exp\left(\frac{2}{\hbar}\int_0^b \{2\mu[V(r) - E]\}^{1/2}\right) , \qquad (2)$$

where the integral is taken over the "forbidden" region IV. The turning points for an incident particle of energy E are defined by $E = V(b) = V(-b)$. If $V(r)$ is taken to be the Coulomb barrier in a cold fusion problem, we find that θ is a very

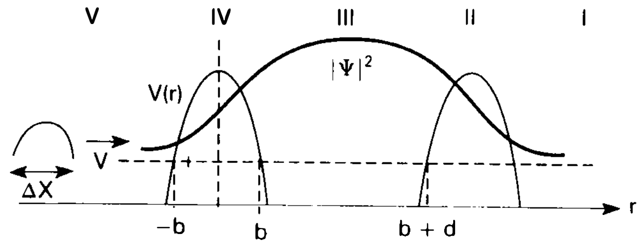

Fig. 1. A pair of potential barriers are shown together with a qualitative representation of the resonant wave function and an incident wave packet.

large number. This ensures the applicability of the WKB approximation. In applying the formulas of Ref. 1, it is therefore sufficient to consider only the leading power of θ.

This definition of the barrier penetration factor θ is consistent with the usage in Ref. 1. Note that the nomenclature may differ between different authors, so that θ^{-1} or θ^{-2} is sometimes called "the barrier penetration factor."

The size of the barrier penetration factor θ has been widely discussed in the cold fusion literature (see, e.g., Refs. 7 and 8). We do not repeat these results, but we consider a few illustrative examples. For the case of a deuteron beam incident on a bare deuteron target, where the beam energy is selected to correspond to room temperature ($E = k_B \times 300$ K = 0.026 eV), we get a barrier penetration factor of $\theta = 10^{3800}$. This value is dramatically reduced if the screening effects of a solid-state environment are taken into account. An optimistic choice of parameters can give barrier penetration factors as small as $\theta \sim 10^{100}$ (Ref. 8).

The transmissivity T is defined as the ratio of the transmitted current to the incident current. For a single barrier, we get $T = \theta^{-2}$. The transmissivity of the two-barrier system in Fig. 1 is to the leading order of θ (Ref. 1)

$$T = \left\{1 + 4\theta^4 \sin^2\left[\frac{1}{2}\left(\pi - \frac{J}{\hbar}\right)\right]\right\}^{-1} , \quad (3)$$

where

$$J = 2\int_b^{b+d} p\,dr . \quad (4)$$

The integrand in Eq. (4) is the momentum in the well (region III). We note that T is typically θ^{-4}, as expected for two barriers penetrated independently in succession. Strong transmission occurs, however, within a set of narrow energy bands since $T = 1$ for $J = \pi(2n+1)\hbar$ (n being 0,1,2...). Following Bush,[4] we introduce an effective wavelength λ according to $J = 4\pi\hbar d/\lambda$, where d is the length of the well between the barriers. The condition for transmission resonance can then be written as $d = (2n+1)\lambda/4$.

Resonant transmission is related to the existence of a metastable state. This state can be viewed as an almost bound particle that is bouncing back and forth in the well (region III) until it eventually escapes. The relative density of particles in the well is[1]

$$\frac{|\Psi_{III}|^2}{|\Psi_V|^2} = 2\theta^2\left\{1 + 4\theta^4 \sin^2\left[\frac{1}{2}\left(\pi - \frac{J}{\hbar}\right)\right]\right\}^{-1} , \quad (5)$$

where Ψ_V is the incident wave. Away from the resonance, there is only a small fraction $\sim\theta^{-2}$ of the incident deuterons that penetrates the first barrier. At the resonance, where the energy of the incident particles matches the energy of the metastable state, we find a very high amplitude in the well. The population of the metastable state at resonance is

$$\int_b^{b+d} |\Psi_{III}|^2\,dr \approx 2\theta^2 d|\Psi_V|^2 . \quad (6)$$

The magic of resonant transmission through a system of high and wide potential barriers is now readily understood. An incident unit wave generates at resonance a huge density of $\sim\theta^2$ in the well. A fraction $\sim\theta^{-2}$ of this density penetrates the second barrier to give a unit transmitted wave in region I. The incident current is partly reflected in the first barrier, but the reflected wave is interfering destructively with the current that leaks from the well back to region V. The smooth passage of current through the barrier system is thus caused by an immense reservoir of particles, hidden in the valley between the barriers. The normal barrier penetration suppression factors are, however, operating even at resonance.

We are now ready to consider cold fusion models based on the transmission resonance concept. In Bush's model,[4] the incident deuterons are considered to move in one dimension, and each barrier in Fig. 1 is taken to be the screened Coulomb potential of a target nucleus at the center of the barrier. Hence, we get a fusion rate that is proportional to the density of mobile deuterons at the center of the barrier. The dominant contribution to this density comes from the penetrating tail of the metastable state. We obtain a fusion rate of

$$\lambda = |\Psi_{III}|^2 K\theta^{-1} = \rho K \exp\left(-\frac{2}{\hbar}\int_0^b \{2\mu[V(r) - E]\}^{1/2}\right) , \quad (7)$$

where $\rho = |\Psi_{III}|^2$ is the density of the metastable state, and E is the resonant energy. This formula is obviously identical to the conventional WKB result in Eq. (1). To get a large fusion yield in transmission resonance models, we are hence still faced with the problem of getting a sufficiently dense and energetic deuteron population within the environment of a deuterated metal. We conclude that the fusion rate is not increased above the WKB estimate even if the transmission resonance model resonance condition is perfectly fulfilled.

The population of the metastable state in a potential well of a real deuterated metal is limited by the electromagnetic repulsion of the deuterons. The lattice is distorted by a too large local concentration of electric charge. This space-charge constraint must be considered in transmission resonance models.

As an example, consider a potential configuration, as in Fig. 1, with a realistic metastable population of one deuteron: $d|\Psi_{III}|^2 \approx 1$. Using Eq. (5), we find that the density of the incident wave is $|\Psi_V|^2 \approx \theta^{-2}d^{-1}$, corresponding to an incident current of $I \approx v\theta^{-2}d^{-1}$. We assume that $d = 1$ Å and that the incident velocity is $v = 1.6 \times 10^5$ cm/s, corresponding to a deuteron energy of $E = 0.026$ eV. The barrier penetration factor is taken to be $\theta = 10^{100}$ as before. The incident current can now be calculated to be $I \approx 10^{-187}$ s^{-1}. Let us for a moment assume that all the current that flows through the barriers is absorbed by the fusion reaction. This is the most optimistic assumption one can make in transmission resonance models of cold fusion, but the resulting fusion rate of 10^{-187} s^{-1} per deuteron is much smaller than both the experimental claims and the dominant contribution to the fusion yield that is caused by the occupation of the metastable state.

RESONANT WAVE PACKET

A particle in a metastable state travels $\sim\theta^2$ times across the well before it escapes. A wave representing the particle must remain in the correct phase with respect to the incident wave while traveling a distance of $\sim\theta^2 d$. The resonance is hence very narrow if θ is very large. The half-width of the transparent energy range is, according to Bohm,[1]

$$\Delta E = \frac{\hbar}{\tau_0}\theta^{-2} \ , \tag{8}$$

where $\tau_0 = 2d/v$ is the time needed for a confined particle to travel across the well and return. The confinement time of a particle in the metastable state is accordingly

$$\Delta t = \tau_0 \theta^2 \ . \tag{9}$$

As an example, consider again a well size of $d = 1$ Å and a resonant energy corresponding to room temperature $E =$ 0.026 eV. We get a single passage time of $\tau_0 = 10^{-13}$ s. Assuming $\theta = 10^{100}$, we get a metastable state lifetime of $\Delta t \sim 10^{187}$ s. This time is much longer than the age of the universe ($\sim 10^{17}$ s).

It is instructive to consider a wave packet incident on a barrier system (see Fig. 1). We assume that the initial wave packet is centered at a resonant energy E. To be transmitted, it must have a width corresponding to ΔE of Eq. (8). The momentum spread is hence $\Delta p = \hbar\theta^{-2}/2d$. The corresponding minimum length of the wave packet is, according to the uncertainty relation,

$$\Delta x = 2d\theta^2 \ . \tag{10}$$

Continuing the numerical example given above, we get $\Delta x \sim 10^{190}$ m, which is much larger than the size of the universe!

The time required for the entire wave packet to pass the first barrier and enter the well is

$$\Delta x/v = \tau_0\theta^2 = \Delta t \ .$$

While the wave packet is in the well, it is reflected many times between the inner walls, and the transmitted part of the wave packet appears in region I after a delay of about $\Delta t = \tau_0\theta^2$, which is the lifetime of the resonant state.

The total fusion yield during the passage of the wave packet comes from the occupation of the metastable state for a time Δt and is well described by integrating the WKB fusion rate [see Eq. (1) or (7)] over the occupation time. We conclude hence that the fusion yield, under transmission resonance conditions, is trivially given by the WKB approximation.

DISCUSSION

Worledge has pointed out the long lifetime of the metastable state in transmission resonance model (see Ref. 24 of Ref. 4). Bush[4] proposed "a way around" that problem. He argues that the incident wave can rapidly pass through the first barrier if the resonance condition is fulfilled. The problem is, according to Bush, that particles inside the well would take a very long time to get through the second barrier. Bush claims that this problem is solved if the double-barrier structure is replaced by a long chain of barriers, so that a particle inside the first well can pass quickly to the second well, thus ensuring a rapid propagation of particles along the chain.

By studying a resonant wave packet, we have, however, demonstrated that the time required to absorb a particle into the well is very long and equal to the decay time of the metastable state. This conclusion is unavoidable since Schrödinger's equation is unchanged by time reversal. Worledge's argument is hence valid for a long chain of barriers as well as for a single pair of barriers.

We have discussed the fusion rate in the proposed one-dimensional transmission resonance model,[4] where the transmitted current is constrained to flow directly through the target nuclei. It is important to note that the deuterons behave quite differently in three dimensions. The deuteron current flows mostly through the potential valleys as far from the nuclei as possible. The fusion yield comes from quite unlikely deuteron trajectories through the target nuclei. Transmission resonances can increase deuteron mobility but cannot directly influence the fusion rate, which presumably is well approximated by Eq. (1).

This technical note shows that the transmission resonance model fails to explain the reported anomalous fusion rates. The intention is not, however, to exclude the possibility that new physical phenomena or subtle coherence effects could give rise to nuclear reaction rates in excess of the conventional WKB approximation. Cold fusion experiments should be judged on their own merits independent of the theoretical debate.

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