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COULOMB BARRIER TRANSMISSION RESONANCE FOR ASTROPHYSICAL PROBLEMS

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In estimating the nonresonance nuclear reaction cross sections $\sigma(E)$ at low energies ($\lesssim$20 keV) needed for astrophysical calculations, it is customary to extrapolate higher energy ($\gtrsim$20 keV) data for $\sigma(E)$ to low energies using the Gamow transmission coefficient representing the probability of bringing two charged particles to zero separation distance, which is unphysical and unrealistic since the Coulomb barrier does not exist inside the nuclear surface. We present a general extrapolation method based on a more realistic barrier transmission coefficient, which can accommodate simultaneously both nonresonance and resonance contributions.

The experimental results from 1968 to 1986 from the ^{37}Cl neutrino detector (the world's only solar neutrino detector in that period) in the Homestake Mine[1] initiated one of the most puzzling and long-lasting problems of modern physics, known as the missing solar neutrino flux problem or, more simply, the solar neutrino problem. The processes $p\ + {}^7\mathrm{Be} \rightarrow {}^8\mathrm{B} + \gamma$ and ${}^8\mathrm{B} \rightarrow {}^8\mathrm{Be}^* + \mathrm{e}^+ + \nu_\mathrm{e}$ ($<$ 15 MeV) produce neutrinos to which the ^{37}Cl detector[1,2] at Homestake Mine is sensitive. The average total rates of solar neutrino (electron type, ν_e) interactions $R^{\mathrm{Cl}}_{\nu_\mathrm{e}}$ (exp) have been measured there by means of the reaction $\nu_\mathrm{e} + {}^{37}\mathrm{Cl} \rightarrow \mathrm{e}^- + {}^{37}\mathrm{Ar}$ from 1970 to the present. More recently, a real time, directional solar–neutrino signal has been observed in the water Cherenkov detector Kamiokande-II (KAM-II)[3] which is sensitive mostly to ${}^8\mathrm{B} \rightarrow {}^8\mathrm{Be}^* + e^+ + \nu_\mathrm{e}$ ($<$ 15 MeV). Of the many experiments that have been conducted, the experimental neutrino deficit is a factor of 2–3 times lower than the accepted prediction from the standard solar model (SSM).[4–6] The much newer ^{71}Ga detectors in the Gran Sasso Laboratory in Italy (GALLEX collaboration)[7] and for the Soviet–American Gallium Experiment (SAGE) at Baksan in the former Soviet Union[8,9] have better detection efficiency. Although the new detectors are lessening the deficit, it has not disappeared.

The SSM has been successful in relating the mass and composition of the sun to its luminosity and lifetime. The SSM has also been widely accepted as it appears to be based upon well-understood nuclear physics. However, this has included approximations that are inconclusively established both for higher energies and for the solar energy regime. In fact, the SSM has appeared to work so well that the preponderance of attempted theoretical solutions has been directed at the neutrinos, rather than the nuclear physics input for the SSM. Of the many proposed hypotheses for solving the solar neutrino problem, neutrino oscillation hypothesis appears to be the most popular.[6,10] However, it is also desirable to re-examine accuracies of the nuclear physics input.

The solar neutrino flux is calculated using low-energy nuclear fusion cross sections $\sigma(E)$ as input data. Since $\sigma(E)$ at solar energies ($\lesssim$ 20 keV) cannot be measured in the laboratory, they are extracted from the laboratory measurements of $\sigma(E)$ at higher energies by an extrapolation procedure based on nuclear theory. However, the energy dependence of the nuclear reaction cross section $\sigma(E)$ cannot be obtained rigorously from first principles, since the many-nucleon scattering problem cannot be solved exactly even if the nucleon–nucleon force is given. Therefore, one must rely on physically reasonable model-dependent parameterization procedure based on a barrier transmission model (BTM). Such a procedure has been used extensively in astrophysical problems[11] involving the Gamow transmission coefficient for the Coloumb barrier.[12,13] In this paper, we present a more general and realistic barrier transmission model which can accommodate simultaneously both nonresonance and resonance contributions for extrapolating $\sigma(E)$ to lower energies.

Previous low-energy (< 20 keV) $\sigma(E)$ for nonresonance reactions involving charged particles used in the standard solar model calculations[4,14] are calculated by extrapolating the experimental values of $\sigma(E)$ at higher energies using the parameterization[11]

$$\sigma(E) = \frac{S(E)}{E} T_{\mathrm{G}}(E) , \tag{1}$$

where $T_{\mathrm{G}}(E) = \exp[-(E_{\mathrm{G}}/E)^{1/2}]$, $E_{\mathrm{G}} = (2\pi\alpha Z_1 Z_2)^2 \mu c^2/2$ with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$, and E is the center-of-mass (CM) kinetic energy. The transmission coefficient ("Gamow" factor) $T_{\mathrm{G}}(E)$ results from the approximation $E \ll B$ (Coulomb barrier height), representing the probability of bringing two charged particles to zero separation distance. This implies that the Coulomb barrier $Z_1 Z_2 e^2/r$ also exists inside the nuclear surface of radius R, which is unphysical and unrealistic. In order to accommodate more realistic transmission coefficients, we write a more general parameterization for $\sigma(E)$ as

$$\sigma(E) = \frac{\tilde{S}(E)}{E} T(E) , \tag{2}$$

where $T(E)$ is the new transmission coefficient for the case in which the fusing system is assumed to have an interior square-well nuclear potential and an exterior Coulomb repulsive potential:

$$V(r) = \begin{cases} -V_0, & r < R , \\ Z_1 Z_2 e^2/r, & r \geq R . \end{cases} \tag{3}$$

For the potential described by Eq. (3), a general solution for the exterior wave function in the exterior region ($r \geq R$) is given by[13]

$$u_l^{\text{ext}}(r) = au_l^{(-)}(r) + bu_l^{(+)}(r)\,, \tag{4}$$

where

$$u_l^{(+)}(r) = e^{-i\delta_l^c}[G_l(r) + iF_l(r)]\,. \tag{5}$$

δ_l^c is the Coulomb phase shift and $u_l^{(-)}$ is the complex conjugate of $u_l^{(+)}$. F_l and G_l are the regular and irregular Coulomb wave functions normalized asymptotically ($r \to \infty$) as

$$\begin{aligned} F_l(r) &\approx \sin[kr - l\pi/2 - \eta\ln(2kr) + \delta_l^c]\,, \\ G_l(r) &\approx \cos[kr - l\pi/2 - \eta\ln(2kr) + \delta_l^c]\,, \end{aligned} \tag{6}$$

where η is the Sommerfeld parameter, $\eta = Z_1Z_2e^2/\hbar v$. For simplicity, our discussion will be limited to the s-wave case, $l = 0$. Generalization to higher partial waves is straightforward.

For the interior region ($r \leq R$), a general solution for the interior wave function is

$$u^{\text{int}}(r) = e^{-iKr} + ce^{+iKr}\,, \tag{7}$$

where $\hbar^2K^2/2\mu = V_0 + E$ with $E = \hbar^2k^2/2\mu$. We introduce two real parameters τ and ϕ and write $c = \tau e^{i\phi}$, $\tau \leq 1$.

Using the boundary condition at $r = R$ (i.e. matching the logarithmic derivatives of Eqs. (4) and (7)), we obtain the barrier transmission coefficient $T(E) = 1 - |b/a|^2$:

$$T(E) = \frac{4s_0\bar{K}_1R}{|(\Delta_0 + is_0) - (\bar{K}_2R - i\bar{K}_1R)|^2}\,, \tag{8}$$

where

$$s_0 = R[(G_0F_0' - F_0G_0')/(G_0^2 + F_0^2)]_{r=R}\,, \tag{9}$$

$$\Delta_0 = R[(G_0G_0' + F_0F_0')/(G_0^2 + F_0^2)]_{r=R}\,, \tag{10}$$

$$\bar{K}_1(E,\,\tau,\,\phi) = \frac{K(1-\tau^2)}{1 + 2\tau\cos(2KR + \phi) + \tau^2}\,, \tag{11}$$

and

$$\bar{K}_2(E,\,\tau,\,\phi) = \frac{-2K\tau\sin(2KR + \phi)}{1 + 2\tau\cos(2KR + \phi) + \tau^2}\,. \tag{12}$$

$T(E)$ in Eq. (8), described by four parameters V_0, R, τ, and ϕ, contain both nonresonance and resonance contributions, and also the interference term between them. The four parameters can be determined from the cross section containing both a resonance part (resonance energy and width) and a nonresonance background.

$T(E)$ has a Breit–Wigner form when $(\Delta_0 + is_0) - (\bar{K}_2R - i\bar{K}_1R) = 0$ at a pole $E = E_R - i\Gamma/2$ in the complex E plane. The resonance energy E_{R} and width Γ are determined by the parameters τ and ϕ for fixed values of V_0 and R. The resonance behavior of $T(E)$, generated from fitting $\sigma(E)$ with particular values of parameters, is a Coulomb barrier transmission (CBT) resonance due to an interplay of Coulomb barrier and nuclear interaction, and is to be distinguished from the conventional resonances such as narrow neutron capture resonances, which are primarily due to the nuclear interaction. The resonances present in $\sigma(E)$, which are shown by some related experiments to be of non-CBT type, are to be treated by conventional methods. Very broad resonance behaviors for cross sections observed in many of the nuclear reactions[15] such as for reactions $^2\mathrm{H(D,p)}^3\mathrm{He}$, $^2\mathrm{H(D,n)}^3\mathrm{He}$, $^3\mathrm{He(D,p)}^4\mathrm{He}$, and $^3\mathrm{H(D,n)}^4\mathrm{He}$ may correspond to CBT resonances and may yield different low-energy extrapolations from those obtained by the use of the conventional transmission coefficient, $T_G(E)$, since the low-energy tail of the CBT resonance is expected to be different from that of the conventional case.

For the case of nonresonance cross section, $\tau = 0$, and $T(E)$ (Eq. (8)) reduces to the result given by Blatt and Weisskopf:[13]

$$T_{\mathrm{BW}}(E) = \frac{4s_0 KR}{\Delta_0^2 + (s_0 + KR)^2}\,. \tag{13}$$

It should be noted that $T_{\mathrm{BW}}(E)$ in Eq. (13) does not have a resonance structure while $T(E)$ in Eq. (8) does.

In the previous parameterizations of $\sigma(E)$, the resonance part of $\sigma(E)$ is parameterized with Breit–Wigner resonance formula to be subtracted from the experimental data[11,16] or included in $S(E)$ in Eq. (1).[15] The nonresonance formula, Eq. (1), is then used to fit the resultant "data". Our more general formula for $T(E)$, Eq. (8), with Eq. (2) will allow us to parameterize the experimental data exhibiting the CBT resonance behavior by the same formula, Eq. (2), thus, avoiding separate use of Breit–Wigner formula for subtracting the resonance contribution from $\sigma(E)$. Furthermore, the interference term between the resonance and nonresonance contributions is automatically included in Eqs. (2) and (8).

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