THE ROLE OF THE LOW-ENERGY PROTON-DEUTERON FUSION CROSS SECTION IN PHYSICAL PROCESSES

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COLD FUSION
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We calculate the proton-deuterium (p-D) fusion reaction rate at low energies ($E \leq 2$ keV in the center-of-mass frame) for a Maxwell-Boltzmann velocity distribution and compare it to those for other reactions involving hydrogen isotopes. It is shown that p-D fusion dominates competing reactions for $E \leq 8$ eV in the center-of-mass frame. The implications for various physical processes are discussed.

I. INTRODUCTION

Historically, proton-deuterium (p-D) fusion or the radiative p-D capture reaction $D(p,\gamma)^3He$ was found to be of astrophysical significance due to its primary role in the proton-proton nucleosynthesis chain.[1,2] The earliest encounter with this reaction in the laboratory was in 1939 by Curran and Strothers,[3] who bombarded heavy ice with low-energy protons but were not very successful in the detection of the emitted radiation. Subsequent experiments involving this reaction suggested that the dominant reaction mechanism was due to direct radiative capture of p-wave protons to the spatially symmetric ground state of ^{3}He via electric dipole emission. The lowest proton kinetic energy for which the cross section for $D(p,\gamma)^3He$ has been measured is $E_p \cong 24$ keV in the laboratory frame or $E_{p\text{-D}} \cong 16$ keV in the center-of-mass (CM) p-D frame.[4] There have been neither measurements nor rigorous theoretical calculations of the cross section $\sigma(E)$ for $D(p,\gamma)^3He$ with $E_{p\text{-D}} < 16$ keV. Reliable values of $\sigma(E)$ at low energies are needed to understand and/or resolve the origin of the mantle heating in the earth,[5] excess heat radiation from the outer planets,[5-9] the solar neutrino problem,[10-12] and other stellar and astrophysical processes.[13,14]

In this technical note, we investigate the role of p-D fusion in recent electrolysis fusion experiments[15-19] using the conventional extrapolation method[13,14] for the fusion cross sections. In Sec. II, we give definitions and expressions for the cross sections and reaction rates of p-D fusion and other competing fusion reactions. In Sec. III, the calculated p-D and other fusion rates are presented and compared. We discuss the implications of the conventionally extrapolated low-energy p-D cross section and reaction rate, as well as that of its large degree of uncertainty, on physical processes. Finally, Sec. IV contains a summary.

II. THEORETICAL FORMULATION

For p-D fusion, the reaction channel is radiative p-d capture:

$$p + D \rightarrow {}^3He + \gamma(5.5 \text{ MeV}) \ . \tag{1}$$

Because of the complexity of the three-nucleon system, no rigorous theoretical calculations of the p-D fusion rate have been carried out at low positive energies below 1.5 keV, although it is now possible to do so employing a modern theoretical formulation with the use of supercomputer facilities.[20] Since there are no direct measurements of the p-D fusion cross section $\sigma(E)$ for reaction (1) with $E_{p\text{-D}} < 16$ keV (CM), we adopt an extrapolation method that has been used in astrophysical calculations[13,14] employing the measured values of $\sigma^{(1)}(E)$ for $E_{p\text{-D}} \geq 16$ keV.

For geophysical and astrophysical applications, the use of a Maxwell-Boltzmann velocity distribution for protons and deuterium with the temperature term kT is appropriate. For the recent electrolysis fusion experiments, the velocity distributions of deuterium and protons (present as an ~0.1% impurity in D_2O) are not known; we assume a Maxwell-Boltzmann distribution without any cutoff for high-velocity components. It has been shown that the results obtained with a cutoff are similar to those without, if a reasonable high-energy cutoff is included in the velocity distribution.[21] The temperature term kT is replaced by the "average" kinetic energy $E_{p\text{-D}}$ in the CM p-D frame.

For a Maxwell-Boltzman velocity distribution, the p-D reaction rate $R_{p\text{-D}}$ ($cm^3 \cdot s^{-1}$) for reaction (1) is given by

$$R_{p\text{-D}} = n_p n_D \langle \sigma v \rangle \ , \tag{2}$$

with

$$\langle \sigma v \rangle = \frac{(8/\pi)^{1/2}}{M^{1/2}(E_{p\text{-D}}^{3/2})} \int \sigma(E) E \exp(-E/E_{p\text{-D}}) \, dE \ , \tag{3}$$

where n_p and n_D are the proton and deuteron number densities, respectively.

For comparison with other fusion reactions involving deuterium, we define the p-D fusion rate Λ (s^{-1}/D) as

$$\Lambda = n_D \langle \sigma v \rangle \ , \tag{4}$$

where $\langle \sigma v \rangle$ is given by Eq. (3) and n_D is assumed to be $\sim 6 \times 10^{22}$ cm^3.

The cross section $\sigma(E)$ in Eq. (3) is parameterized as (E in the CM frame)

$$\sigma(E) = \frac{S(E)}{E} \exp[-(E_G/E)^{1/2}] \ , \tag{5}$$

which is the conventional form based on the assumption that reaction (1) is a nonresonant charged-particle reaction. The Gamow energy E_G is given by $E_G = (2\pi\alpha Z_p Z_D)^2 Mc^2/2$ with reduced mass $M = m_p M_D/(m_p + M_D) \cong \frac{1}{3}M_D$. For reaction (1), $E_G^{1/2} = 25.64\ (\text{keV})^{1/2}$.

With a Taylor series expansion of $S(E)$ as given in Ref. 13,

$$S(E) \approx S(O) + S'(O)E + \frac{1}{2} S''(O)E^2 \ , \tag{6}$$

the integral in Eq. (3) can be solved approximately[21] to yield (in an asymptotic series, valid for $E_{p\text{-D}} \ll E_G$)

$$\langle \sigma v \rangle = \left(\frac{2}{M}\right)^{1/2} \frac{\Delta E_0}{(E_{p\text{-D}})^{3/2}} S_{eff} \exp(-\tau) \ , \tag{7}$$

where

$$\Delta E_0 = 4(E_0 E_{p\text{-D}}/3)^{1/2} \ ,$$

$$\tau = 3E_0/E_{p\text{-D}} \ ,$$

and

$$S_{eff} = S(O)\left[1 + \frac{5}{12r} + \frac{S'(O)}{S(O)}\left(E_0 + \frac{35}{36}E_{p\text{-D}}\right) + \frac{1}{2}\frac{S''(O)}{S(O)}\left(E_0^2 + \frac{89}{36}E_0 E_{p\text{-D}}\right)\right] \ ,$$

with

$$E_0 = (E_G^{1/2} E_{p\text{-D}}/2)^{2/3} \ .$$

The average kinetic energy $E_{p\text{-D}}$ (CM) for the various physical processes under consideration is expected to be ~0.1 to 2000 eV.

In a typical electrolysis experiment, there are several possible mechanisms that can accelerate D^+ ions through D_2 gas bubbles or through a D_2 gas layer that has formed on the surface of the cathode. Although the potential difference across a given D_2 gas bubble is expected to be a fraction of the applied potential across the electrolysis cell, Rabinowitz and Worledge[22] suggested several nonequilibrium situations in which large potential differences can occur. First, H_2/D_2 gas bubbles can form a layer that virtually covers the palladium cathode, thereby allowing the formation of a high double-layer electric field that is enhanced at the sharp tips of surface asperities (or whiskers); this electric field can be $\sim 10^9$ V/m with a high D^+ current, even though the voltage across the electrolysis cell is only ~1 V. Second, the breaking of direct contact between the electrolyte solution and small regions on the surface of the cathode by the interposition of accumulating D_2 gas bubbles leads to the possibility of breakdown arcing across these bubbles with attendant huge spark discharge current densities ($\lesssim 10^5$ A/cm^2) (Ref. 23); this would be especially true in the presence of asperities. Third, a small number of deuterons can become entrained with high current density electrons. Even though these electrons can have energies only of the order of ~10 eV, the entrained D^+ can obtain energies of ~37 keV, since the ratio of the deuteron energy to electron energy would be $(M_D v^2/2)/(M_e v^2/2) = M_D/M_e \approx 3670$.

For comparison with other fusion processes involving protons and deuterons, we consider the following fusion reactions:

$$\text{D} + \text{D} \rightarrow {}^3\text{H}(1.01\ \text{MeV}) + p(3.02\ \text{MeV}) \ , \tag{8}$$

$$\text{D} + \text{D} \rightarrow {}^3\text{He}\ (0.82\ \text{MeV}) + n(2.45\ \text{MeV}) \ , \tag{9}$$

$$\text{D} + \text{D} \rightarrow {}^4\text{He} + \gamma(23.8\ \text{MeV}) \ , \tag{10}$$

$$p + \text{T} \rightarrow {}^4\text{He} + \gamma(19.8\ \text{MeV}) \ , \tag{11}$$

and

$$\text{D} + \text{T} \rightarrow {}^4\text{He}(3.52\ \text{MeV}) + n(14.07\ \text{MeV}) \ . \tag{12}$$

For the above reactions, $E_G^{1/2}$ is given by 31.39, 31.39, 31.39, 27.19, and 34.38 (keV)$^{1/2}$, respectively. The corresponding fusion rates are calculated with Eqs. (3), (4), and (5) using the appropriate values of $S(E)$ given in Ref. 13 and replacing $E_{p\text{-D}}$ and E_G with the CM energy and Gamow energy, respectively, for each reaction.

III. FUSION RATES AND PHYSICAL PROCESSES

The extrapolated cross section $\sigma(E)$ for the p-D fusion reaction (1) calculated from Eqs. (5) and (6) using parametric values from Ref. 13 is shown as a function of $E_{p\text{-D}}$(CM) in Fig. 1 ($E_{p\text{-D}} \leq 20$ eV) and Fig. 2 ($E_{p\text{-D}} \leq 2$ eV). The extrapolated values of $\sigma(E)$ for reactions (1) and (8) through (12) are used to calculate the equivalent fusion rates as a function of the CM energy E. The calculated values for these fusion rates are plotted for comparison in Fig. 3 [E(CM) $\leq$ 2 eV], Fig. 4 [E(CM) $\leq$ 20 eV], and Fig. 5 [E (CM) $\leq$ 2 keV].

For the extracted neutron production rate of $\Lambda_{exp}^n \approx 10^{-23}\ \text{s}^{-1}\cdot\text{D}^{-1}$ by Jones et al.,[5] the actual total observed rate is $R_{exp}^n = (4.1 \pm 0.8) \times 10^{-3}\ \text{s}^{-1}/\epsilon \approx 0.4\ \text{s}^{-1}$ with a neutron detection efficiency of $\epsilon = 0.01$. When we consider the surface fusion mechanism[24,25] for electrolysis experiments,[15-20] the total reaction rate R_{calc}^n, which takes into account a velocity distribution $f(v)$, is given by

$$R_{calc}^n = (\Phi/\bar{v}) \int f(v) v P(v)\, dv \ , \tag{13}$$

where v is the relative D^+ velocity, $f(v)$ is normalized to $\int f(v)\,dv = 1$, $\bar{v} = \int v f(v)\,dv$, and $\Phi = n_i A \bar{v}$ for the incident D^+ density n_i and the target area A. The value $P(v)$ is the probability of a deuteron undergoing a fusion reaction while slowing down in the deuterated palladium target. For order of magnitude estimates, Eq. (13) can be approximated by

$$R_{calc}^n \approx (\Phi/\bar{v}) \Delta x \Lambda(E) \ , \tag{14}$$

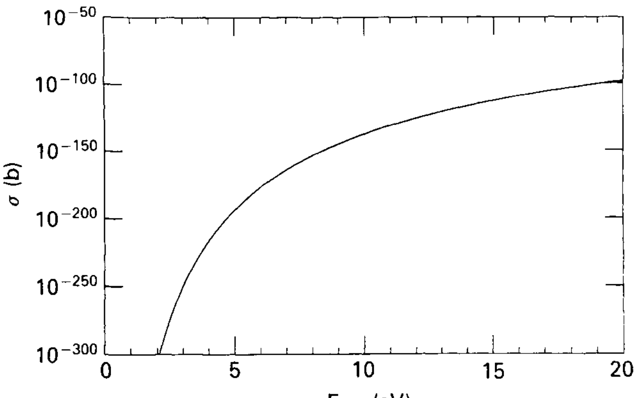

Fig. 1. The extrapolated cross section for reaction (1) for $E_{p\text{-D}} \leq$ 20 eV.

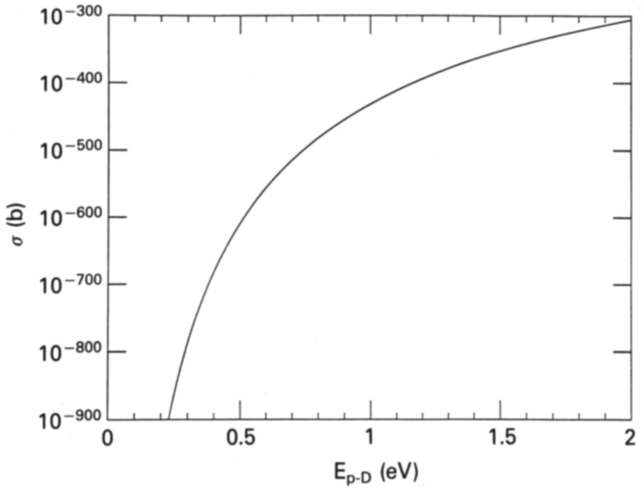

Fig. 2. The extrapolated cross section for reaction (1) for $E_{p\text{-D}} \leq$ 2 eV.

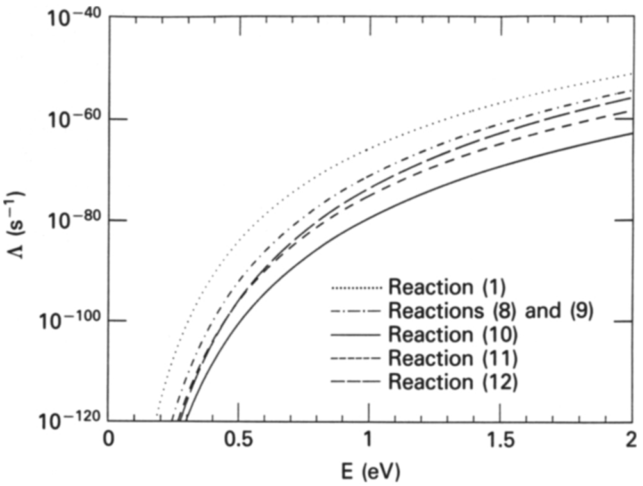

Fig. 3. Calculated reaction rate for *E*(CM) ≤ 2 eV.

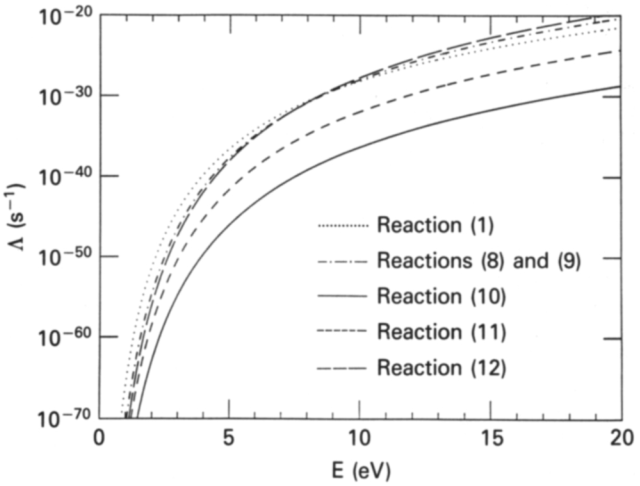

Fig. 4. Calculated reaction rate for *E*(CM) ≤ 20 eV.

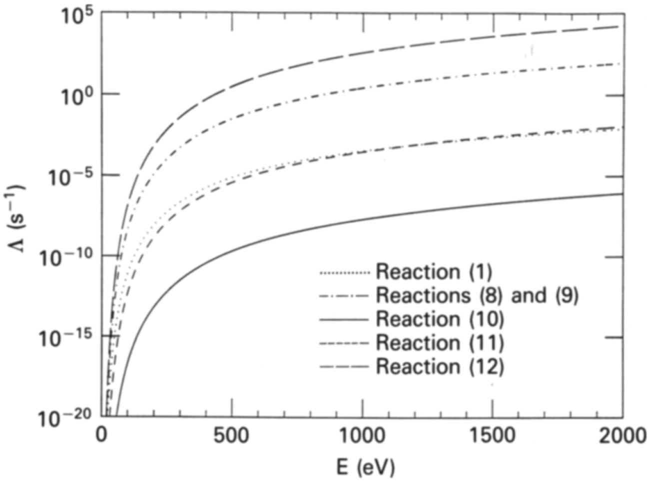

Fig. 5. Calculated reaction rate for *E*(CM) ≤ 2 keV.

where Δx is the effective interaction thickness (~10 Å) of the target and $\Lambda(E)$ is given by Eq. (4). For the calculated value of $\Lambda^n_{calc} \approx 10^{-20}$ s^{-1} for $E_{\text{D-D}} \approx 20$ eV ($\bar{v} = 2.5 \times 10^6$ cm/s) from Fig. 4, $R^n_{calc} \approx 2.5 \times 10^{-11}$ for a net spark discharge current[23] of $\Phi \sim 10^4$ A [which is not the same as the measured external current (~0.1 A)]. The remaining discrepancy of 10^{10} between $R^n_{exp} \approx 0.4$ s^{-1} and $R^n_{calc} \approx 2.5 \times 10^{-11}$ s^{-1} may be due to the electron screening effect (which may reduce the discrepancy by a factor of $\lesssim 10^8$) and/or due to the inadequacy of the extrapolation formula, Eq. (5), at these low energies (see below and Ref. 26).

As can be seen from Figs. 3 and 4, the *p*-D fusion rate becomes larger than other fusion rates for *E*(CM) ≲ 8 eV. This has several physically significant consequences. At $kT = E_{p\text{-D}} \approx 0.1$ to 0.25 keV [corresponding to $T \approx (1$ to $3) \times 10^3$ K), *p*-D fusion is expected to be the dominant process producing the thermonuclear heating in the mantle. Since $\Lambda_{p\text{-D}}/\Lambda_{\text{D-D}} \approx 10^{12}$ from Fig. 3 and $n_p/n_{\text{D}} \approx 10^4$, we expect $R_{p\text{-D}}/R_{\text{D-D}} \approx 10^{16}$ at $kT \approx 0.1$ to 0.25 eV. For electrolysis fusion experiments, $R_{p\text{-D}} \approx R_{\text{D-D}}$ at *E*(CM) ≈ 2.5 eV since $n_p/n_{\text{D}} \approx 10^{-3}$ and $\Lambda_{p\text{-D}}/\Lambda_{\text{D-D}} \approx 10^3$; i.e., *p*-D fusion is expected to compete with D-D fusion for *E*(CM) < 2.5 eV.

It has been recently suggested[24-26] that the conventional extrapolation method for the cross sections for reactions (8) and (9) may not be reliable and needs to be tested by direct experimental measurements and/or to be justified by rigorous theoretical calculations. Recent results of indirect measurements[27] of the D-D fusion cross section indicate that the extrapolation may not be valid at low energies. If, in the future, experimentally measured values of $\sigma(E)$ for the *p*-D fusion reaction (1) at low energies, $E_{p\text{-D}} \leq 1.5$ keV, turn out to be larger than the extrapolated values shown in Figs. 1 and 2, then the enhanced *p*-D fusion rate can be regarded as (a) a possible explanation of the earth's internal heating[5]; (b) as a source for the excess heat produced by the other planets[5-9]; (c) as a thermonuclear process competing with D-D fusion in electrolysis fusion experiments; and (d) as an alternative explanation of the solar neutrino problem[10-12] in combination with other modified thermonuclear reaction processes at stellar temperatures.

IV. SUMMARY

It is shown that the low-energy p-D fusion rate $\Lambda(E_{p\text{-D}})$ calculated with a Maxwell-Boltzmann velocity distribution is larger than other fusion rates for $D(D,p)^3H$, $D(D,n)^3He$, $D(D,\gamma)^4He$, $^3H(p,\gamma)^4He$, and $^3H(D,n)^4He$ at CM energies below 8 eV. If the conventional extrapolation used turns out to be unreliable[24-26] and underestimates the fusion cross sections as implied by recent indirect measurements[27] of the D-D fusion rate at low energies, the p-D fusion process could provide a plausible explanation for the earth's internal heating.[5] Enhanced cross sections for p-D, D-D, and other fusion processes at solar and stellar temperatures may necessitate a reformulation of previous astrophysical calculations and may lead to a resolution of several unsolved astrophysical problems such as the solar neutrino problem[10-12] and the excess heat radiation from the outer planets.[5-9] Therefore, it is important to test the validity of the extrapolation method, Eq. (5), at low energies by experimental measurements of the cross section for reaction (1) for $E_{p\text{-D}}(\text{CM}) < 16$ keV using the direct reaction (1) or its inverse reaction, $^3He(\gamma,p)D$. Direct measurements of $\sigma(E)$ for $E(\text{CM}) \gtrsim 1.5$ keV (corresponding to the inferred temperature of the solar core) may be possible with currently available experimental techniques and technologies using innovative experimental designs. For $\sigma(E)$ below $E(\text{CM}) = 1.5$ keV, new experimental techniques may be required.

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