

AN ANALYSIS OF COLD AND LUKEWARM FUSION

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COLD FUSION

TECHNICAL NOTE

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Experimental reports continue to suggest that the crystal-line solid state may present a unique environment for deuterium-deuterium (d-d) fusion at ambient temperature (cold fusion). The analysis herein shows that newly reported cluster-impact d-d fusion at energies ~100 eV has much in common with cold fusion and might appropriately be called lukewarm fusion. Both phenomena evidently need a novel theoretical approach for their understanding. A deuteron effective mass approach is proposed as a possible explanation of the reported experimental results.

INTRODUCTION

For historical reasons as well as because of the difficulties encountered by the vast majority of investigators in reproducing or even repeating the reported results, cold fusion will need many further experimental confirmations to solidly establish its scientific basis. Curiously, and in sharp contrast to this, apparently it has taken only one experiment to establish low-energy cluster-impact fusion as scientific fact. Even though the ~100-eV energies of the latter are significantly higher than the ~0.03 eV of cold fusion, we show that they have much in common. In particular, they both represent anomalously high fusion rates based on conventional wisdom. This is obvious with respect to cold fusion, but much less so for cluster fusion and may have contributed to the relative ease with which it was accepted. If "cold fusion" is a proper term for ambient temperature fusion, then the term "lukewarm fusion" is both convenient and appropriate for fusion at ~100 eV.

LUKEWARM FUSION

Enormous Discrepancy

In demonstrating deuteron-deuteron (d-d) fusion rates of $10^{-1}/d \cdot s^{-1}$ at incident deuteron energies of only ~100 eV, Beuhler et al.¹ have made a most significant contribution. They note a discrepancy between their experimentally obtained fusion cross section σ and one that they have calcu-

lated. They say, "By assuming no compression occurs, the experimental results lead to a value of σ more than 10 orders of magnitude larger than that computed for 300 eV D impacts. . . ." Their method of calculation leads to a σ that is 25 orders of magnitude smaller than the value obtained from their experiment. Significantly, even when both compression and electron shielding effects are included, a discrepancy of ~15 orders of magnitude remains.

The theoretical fusion cross section² used by Beuhler et al. is

$$\sigma(E) = [S(E)/E] \exp(-31.28/E^{1/2}), \quad (1)$$

which is based on the experimental astrophysical function $S(E)$ obtained for energy measurements above ~10 keV. Values of S for d-d fusion at low energies^{2,3} are given between 53 and 108 b·keV. For calculating σ and other parameters in this technical note, this range leads only to a factor of 2 in the computed values. Beuhler et al. use $55 \times 10^{-24} \text{ cm}^2 \cdot \text{keV}$, and this value is used in Eq. (1), where E is the energy in the center of mass (CM) system in kilo-electron-volts. For d-d fusion, we calculate the exponent to be 31.4 rather than 31.28, but this would only make a 36% difference in the calculated σ .

For a $(D_2O)_N$ cluster accelerated to an energy E_N , the energy of the individual deuterons in the lab frame is

$$\begin{aligned} E_{lab} &= E_N/N[(2m_D + m_O)/m_D] \\ &= E_N/N[(2m_D + 8m_D)/m_D] \\ &= E_N/10N. \end{aligned} \quad (2)$$

For $E_N = 300 \text{ keV}$ and $N = 100$, $E_{lab} = 300 \text{ eV}$ in agreement with Beuhler et al.

$$\begin{aligned} E &= E_{CM} = [m_{target}/(m_{target} + m_{missile})] E_{lab} \\ &= \left[\frac{1}{2} \right] E_{lab} = 150 \text{ eV} \quad \text{as } m_t = m_m = m_D, \end{aligned}$$

which gives the largest value of E_{CM} with respect to E_{lab} . Since the kinetic energy received from the accelerator is the dominant energy on collision, it is unlikely that the CM energy of the deuterons after impact will significantly exceed 150 eV. The value of E may even be less, as appreciable energy could be transferred to the electron ensemble, even though it is a small amount per electron collision. As N gets larger, E decreases, so most of the clusters have an energy < 150 eV.

Thus, for 150 eV Eq. (1) yields $\sigma = 3.1 \times 10^{-37} \text{ cm}^2$. An

experimental cross section σ_{exp} can be inferred from Beuhler et al.¹ They estimate a fusion rate $N = 10^{-1}/d \cdot s = n\sigma v$, where n is the number density of impacted deuterons and v is the mean velocity. From their value¹ of "0.8 g/cm³, ~5 times the density of liquid D₂ under normal pressure," a normal number density value of $n_0 = 2.4 \times 10^{22}/\text{cm}^3 \sim 10^{22}/\text{cm}^3$ is obtained. An upper limit of the one-dimensional compression yields $n \sim 10n_0 \sim 10^{23}/\text{cm}^3$. For the minimum d - d separation, $2R$, we have $10n_0 \geq (n_0)^{2/3}/2R$, where $(n_0)^{2/3}$ is the two-dimensional number density perpendicular to the collision direction. Thus, $R \geq (\frac{1}{30})(n_0)^{-1/3} \sim 2 \times 10^{-11} \text{ m} = 0.2 \text{ \AA}$. For $E = 150 \text{ eV}$, $v = 1.70 \times 10^7 \text{ cm/s}$. Thus, $\sigma_{exp} = 5 \times 10^{-32} \text{ cm}^2$. This produces a discrepancy of $\sigma_{exp}/\sigma \sim 10^{25}$.

Discrepancy Reduction

Let us determine the extent to which this discrepancy can be resolved or reduced by developing a theoretical approach that incorporates the effects of both compression and electron shielding, since Eq. (1) does not include these. The cross section σ for a nuclear reaction between charged particles can be expressed as a product of four factors:

$$\sigma = \sigma_c G T \rho , \tag{3}$$

where

- σ_c = cross section for collision
- G = coulomb tunneling coefficient
- T = ratio of the net flux of particles transmitted into the nuclear potential well compared to that penetrating the coulomb barrier
- ρ = function of the strong interaction leading to a given fusion reaction (exit channel).

To solve for G , the Schrödinger equation is solved assuming a potential function for the deuterons in a solid. Various models for the screening potential of the free electrons in a solid lead to roughly similar results. The differences in the models may be important when better experimental data are available. At this stage, a model of a spherical shell of radius R of negative charge surrounding each deuteron will suffice even though other models such as a uniform cloud of electrons give higher tunneling and fusion rates. This model leads to a shifted coulomb potential with an analytical solution. The potential energy as a function of radius r outside the nuclear well is

$$V = (e^2/4\pi\epsilon_0) [(1/r) - (1/R)] , \quad r_1 \leq r \leq R , \tag{4}$$

where

- e = deuteron charge
- r_1 = nuclear well radius
- $V = 0$ for $r > R$
- $V = V_n$ (the square well nuclear potential) for $0 \leq r \leq r_1$.

Equation (4) may be interpreted as resulting from the first-order expansion of the exponentially shielded potential $V = (e^2/4\pi\epsilon_0 r) \exp[-r/R]$.

Solution of the Schrödinger equation yields the tunneling coefficient,

$$G = B \exp[-2g(r_1)] , \tag{5}$$

where

$$g(r_1) = (\pi/2\hbar) [(2e^2/4\pi\epsilon_0)\mu r_2]^{1/2} . \tag{6}$$

The classical turning point is r_2 , and μ is the deuteron reduced mass $m_d/2$. The astrophysical S function for d - d fusion of $108 \text{ keV} \cdot \text{b} = 1.73 \times 10^{-42} \text{ J} \cdot \text{m}^2$, which can be combined with Eq. (3) to yield

$$\sigma = (S/E) \exp[-2g(r_1)] . \tag{7}$$

The total fusion rate in number of d - d fusion reaction/cm³·s⁻¹ is

$$N_t = \left(\frac{1}{2}\right) n^2 \sigma v .$$

Thus,

$$N_t = \left(\frac{1}{2}\right) [2n^2 S / (2\mu E)^{1/2}] \exp[-2g(r_1)] . \tag{9}$$

The fusion rate per deuteron is $N = N_t/n$, and the excess power due to fusion is $P = N_t (4.03 \text{ MeV})$ since $d + d \rightarrow \text{H}^3 + \text{H}^1 + 4.03 \text{ MeV}$, assuming only this branch of the d - d reaction occurs. Since N_t , N , and P are all proportional to σ , the same discrepancy occurs for these.

Table I shows the calculated values of N , N_t , and P as functions of the CM energy and one-half the nearest separation of the deuterons in angstroms. As can be seen, only at the highest energy of 1200 eV and the smallest R of 0.02 Å, both of which greatly exceed the likely values of 150 eV and 0.2 Å in these experiments, can the Beuhler et al. value of 10^{-1} fusion/ $d \cdot s$ be achieved. For 1200 eV and 0.05 Å, the calculated value of 10^{-2} fusion/ $d \cdot s$ can be increased to 10^{-1} if the electron screening were due to a spherical cloud rather than a spherical shell of negative charge as calculated by numerical integration. For the more realistic 150-eV and 0.2-Å case, N can be increased from 10^{-20} fusion/ $d \cdot s$ to 10^{-16} by this method.

Ten orders of magnitude could be accounted for by compression and electron screening, but a discrepancy of ~15 orders of magnitude remains ($10^{-1}/10^{-16}$). The analysis shows that the remaining discrepancy can be resolved in terms of proximity and energy only by greatly exceeding reasonable values for these parameters. One may conclude that a conventional approach is not sufficient to explain their fusion rates.

COLD FUSION

Unexpected Fusion

On March 23, 1989, Fleischmann and Pons announced their achievement of fusion at room temperature in a palladium electrolytic cell using heavy water (deuterium oxide) as the electrolyte. The Fleischmann and Pons results⁴ were particularly surprising to the scientific community because there had been no hint of a fusion-type reaction in other circumstances where the same isotopes were involved. Specifically, liquid solutions of deuterium oxide and of tritium oxide have densities comparable to that of the deuterium in the palladium in the Fleischmann and Pons experiment. Fusion has not been seen in these liquids. In addition, palladium has been used to purify hydrogen and its isotopes from other gases as the hydrogen isotopes move readily through windows of palladium, but other gases do not. However, fusion has not been observed in these circumstances either. Therefore, for the fusion to occur at the reported levels in such metals, it appears that hitherto unconsidered physical mechanisms must be present in a crystalline solid that are not present in

TABLE I
Calculated Values of Lukewarm Fusion Rates and Power Density

E_{CM} (eV)	Fusion Rate, N (fusion/ $d \cdot s$)				Total Fusion Rate, N_t (fusion/ $cm^3 \cdot s$)				Power Density, P (W/ cm^3)			
	R (Å)				R (Å)				R (Å)			
	0.2	0.1	0.05	0.02	0.2	0.1	0.05	0.02	0.2	0.1	0.05	0.02
150	10^{-20}	10^{-17}	10^{-12}	10^{-6}	10^4	10^6	10^{11}	10^{18}	10^{-8}	10^{-6}	10^{-1}	10^5
300	10^{-14}	10^{-12}	10^{-9}	10^{-5}	10^9	10^{11}	10^{14}	10^{19}	10^{-3}	10^{-1}	10^2	10^7
600	10^{-8}	10^{-7}	10^{-5}	10^{-2}	10^{15}	10^{17}	10^{18}	10^{21}	10^3	10^4	10^6	10^9
1200	10^{-3}	10^{-2}	10^{-2}	10^{-1}	10^{20}	10^{21}	10^{22}	10^{23}	10^8	10^9	10^9	10^{11}

the liquid. Interestingly, Gilman⁵ in 1971 suggested fusion experiments in the highly loaded solid LiD₂F.

Fusion Ingredients

Four ingredients are essential for nuclear fusion of either the hot or the cold variety:

1. tunneling probability
2. collision frequency
3. fusion probability
4. sustaining the reaction.

The fusion rate is proportional to the product of the first three processes. The ability to achieve fusion at ambient temperature appears to be related most strongly to unexpected increases in tunneling probability and collision frequency in the CM system.

Tunneling Probability

The fusion rate is extremely sensitive to the tunneling probability, which in turn is extremely sensitive to the mass of the tunneling particles and their proximity. Due to the periodic potential of the lattice ions in a solid and the wave nature of the particles that move freely in this lattice, it is possible for the effective mass of the particle in the solid to differ from its mass in free space. Our calculations show that it is possible for the effective mass of the deuterium nuclei in a solid to be sufficiently less than the mass of deuterons in free space to increase the tunneling coefficient by many, many orders of magnitude. Another very important effect of the solid is to bring deuterons much closer together than they could otherwise get at ambient temperature.

Enhancing the Collision Frequency

The fusion rate is also proportional to the collision frequency of the deuterium nuclei. In three dimensions, the collision frequency per particle is

$$F_3 = n\sigma_c v ,$$

where

- n = number density
- σ_c = collision cross section
- v = mean thermal velocity.

We expect this number to be roughly the same in the liquid state and in ordinary solid solution as found in palladium. There may be preferential pathways in a solid that decrease

the degrees of freedom in the solid so that the fusing particle is confined essentially to one- or two-dimensional motion in the solid—that is, the particles may be able to move only in certain channels or planes. Increasing the number density and reducing the dimensionality substantially increase the collision frequency compared with a free-space plasma, and thus greatly enhance the fusion rate.⁶ Channeling increases the probability of a nearly one-dimensional collision, with essentially the absence of angular momentum in the final state. This may permit low-energy resonances, which greatly increase the fusion cross section.

Surface Versus Bulk

One of the most basic questions regarding cold fusion is whether the phenomenon is a bulk effect occurring inside the solid or whether it is a surface effect. In an electrolytic cell, sharp asperities (microprotrusions) can grow on the cathode.⁷ Field enhancement at their tips, together with the already present high double-layer electric field, can lead to very high electric fields $\sim 10^9$ V/m, the emission of electrons, and high current densities even though the voltage across the cell is only ~ 1 V. Bubble production can locally separate and unite the electrolyte from the cathode like the opening and closing of electrical contacts. Arcing occurs during the separation of two metal electrical contacts even though the voltage source is very low (~ 1 V), generating high electric fields and high temperatures as well as a metal vapor plasma. This and other equilibrium or nonequilibrium effects involving inductive temperature amplification ($L di/dt$ transformerlike, producing high energies microscopically) could be a miniature form of hot fusion. One possibility is that a small number of deuterons can become entrained with high current density electrons that have energies ≥ 10 eV. The number of such deuterons would be a small fraction of the number of electrons and would attain about the same velocity as the electrons. The ratio of the energy of the deuterons to the energy of the electrons would be $(M_d v^2/2)/(m_e v^2/2) = M_d/m_e = 3670$. Thus, a small number of deuterons might attain energies of ~ 37 keV.

Surface effects may be able to account for the large variability of results both for a given scientist and among the diversity of investigators. The growth rate of asperities can be a function of which crystallographic planes are at the surface and may be a factor in the fairly long incubation periods for observation of tritium or excess power. Similarly, the method of preparation of the palladium may well affect which crystal face is at the surface. Surface contamination

could either decrease the influx of deuterons into the bulk or just poison a reaction process at the surface.

Fusion in the Solid

General Derivation of Fusion Rate

A very important effect of the solid is to bring deuterons much closer together than they could otherwise be at ambient temperature. Although the average separation of the deuterons is $\sim 1.4 \text{ \AA}$ in heavily loaded palladium, the deuterons can be in equilibrium at a separation as close as 0.94 \AA (Refs. 8, 9, and 10). Closer separation is possible in nonequilibrium processes.

Here we assume the deuterons have a reduced *effective* mass μ^* for $r > R$, where R was defined for Eq. (4). In the nuclear well, the deuterons have a reduced mass: $\mu =$ their *free* reduced mass ($m_d/2$). Inside the barrier (classically the forbidden region), their mass varies continuously from the reduced *effective* mass outside the barrier to the reduced *true* mass inside the nuclear well, as a two-body approximation of the many-body problem. (In the many-body approach, the free mass would apply everywhere.) To obtain an analytic solution, we have μ at r_1 to μ^* at r_2 the classical turning point: $\mu_B \doteq (\mu - \mu^*)(r_1/r) + \mu^*$ for $r_2 \gg r_1$.

Solution of the Schrödinger equation yields the tunneling coefficient as for Eq. (5), but with

$$g^*(r_1) = (\pi/2\hbar) [(2e^2/4\pi\epsilon_0)\mu^*r_2]^{1/2} . \quad (10)$$

The measured value^{2,3} of the astrophysical S function for d - d fusion is $108 \text{ keV} \cdot \text{b} = 1.73 \times 10^{-42} \text{ J} \cdot \text{m}^2$, which can be combined with Eq. (3) to yield

$$\sigma = (S/E)\exp[-2g^*(r_1)] . \quad (11)$$

Thus, Eq. (9) becomes

$$N = [2n^2S/(2\mu^*E)^{1/2}]\exp[-2g^*(r_1)] , \quad (12)$$

where E is the energy in the CM system.

Decreased Effective Mass

We now examine the effects of the crystalline environment on this relationship. These effects address the deuteron effective mass. To our knowledge this is the first time that these concepts have been applied to the fusing particles as a possible explanation for cold fusion. Recent publications have postulated effective mass concepts for the electrons in a solid (as an analog to muon-catalyzed fusion), as well as other mechanisms to account for increased fusion.¹¹⁻¹⁶ Even if the electron concepts are not applicable for a bound system to enhance the fusion rate, our fusing particle (deuteron) effective mass concept may be valid^{6,17} as it is applied outside the barrier where the inertia of the unbound deuterons is determined by the lattice. At the very least, one may consider the following a parametric analysis of what decreased deuteron mass is needed to account for the observed fusion rates.

The periodic potential in which deuterons move in palladium and their interactions with the ionic lattice and its constituents is similar to that of electrons, and the effective mass concept applies to both. A deuteron in such a crystal is subject to forces from the crystal lattice as well as the coulomb force from another deuteron. The Hamiltonian of two deuterons contains contributions from the periodic potential of the lattice, electrons, and from interaction of the two deuterons. Simplification to the two-body (two deuterons) Hamiltonian may be accomplished by using the effective mass for

$r > a$, the lattice spacing. Experiments involving $r > a$ and time $t > a/c$, which consider only external forces, will infer an inertia for the charged particle equal to the effective mass. As nuclear distances are approached ($\sim 10^{-15} \text{ m}$), the interaction between the two deuterons dominates over the lattice contribution, and the free mass is appropriate.

In a region of periodic potential perturbations, charged particles behave dynamically as if they possess an effective mass m^* (less or greater than the free mass) given by

$$m^* = \hbar^2/(d^2E/dk^2) . \quad (13)$$

Just as the electrons see an attractive periodic potential, the deuterons see a repulsive potential at each of the metal ions with further periodic potential perturbations at the positions of the octahedral interstitial sites when essentially all are occupied by deuterons as in the beta-phase PdD_x with $x \geq 0.75$. Although a band calculation is not attempted for the deuteron effective mass m_e , the expected magnitude can be estimated from the simple Kronig-Penney model for a unit charge moving with periodicity, a :

$$m^* \sim \hbar^2/2a^2E , \quad (14)$$

where E are the eigenstate energies for deuterons moving between the interstitial sites. Equation (14) properly estimates the effective mass of electrons in terms of the Fermi energy. For deuterons (bosons of spin 1), even though their energies are low, being distributed around thermal, Eq. (14) gives $m^* \sim 0.01$ times the free deuteron mass as a lower limit. A decreased mass can profoundly increase the G factor. A triton should have an even smaller effective mass than a deuteron because it is a Fermi particle and hence has higher E . Because of this, a prediction of this model is that *heavy loading of the lattice with tritons and deuterons should give even higher fusion rates*. The difference in zero-point amplitude of deuterons, tritons, and protons in palladium may also be significant.

The effects of deuteron effective mass are calculated in Table II where the number of deuteron-deuteron fusion/ $\text{cm}^3 \cdot \text{s}^{-1}$ is calculated for PdD_x for $x \geq 0.75$. The fusion rate is calculated for a series of values of effective mass for E_{CM} of 0.025, 0.15, and 1 eV, as could be found in the high-energy tail of the thermal energy distribution.

The value $E_{\text{lab}} = E_{\text{CM}}$ corresponds to symmetric collisions of deuterons in transit between interstitial sites, which may be more likely in a lattice that provides preferential pathways of motion than in a high-temperature plasma. (It is interesting to note similarities with superconductivity.¹⁸) This is another mechanism that can increase the fusion rate by many orders of magnitude. As Table II shows, fusion rates of $> 10^{10}/\text{cm}^3 \cdot \text{s}^{-1}$ may be obtained by these mechanisms, which may account for the reported excess power and radiation attributed to cold fusion.

In addition to accounting for reported fusion rates,^{4,19-24} this theory further predicts two generic (i.e., for $\mu^* = \mu$) properties of cold fusion in the bulk of a solid:

1. There is an extremely strong dependence on deuteron concentration, as reflected in the results for decreasing R .
2. There is not a strong temperature dependence of the fusion rate right up to the melting point. For palladium this is 1828 K, corresponding to $E = 0.152 \text{ eV}$.

In free space, a large increase in fusion rate would be expected with increased temperature. With a decreased solubility of deuterons in palladium, and hence decreased density as indicated by the large differences between the $R = 0.5, 1.0$,

TABLE II
Calculated Cold $d-d$ Fusion Rates

μ^*/μ	$R = 0.5 \text{ \AA}$			$R = 1 \text{ \AA}$			$R = 1.4 \text{ \AA}$		
	$E_{CM} \text{ (eV)}$			$E_{CM} \text{ (eV)}$			$E_{CM} \text{ (eV)}$		
	0.025	0.15	1.0	0.025	0.15	1.0	0.025	0.15	1.0
1	10^{-45}	10^{-44}	10^{-43}	10^{-80}	10^{-79}	10^{-75}	10^{-102}	10^{-100}	10^{-95}
0.1	10^{11}	10^{11}	10^{12}	10^{-1}	10^{-1}	10^{-1}	10^{-9}	10^{-8}	10^{-6}
0.05	10^{19}	10^{19}	10^{20}	10^9	10^{10}	10^{11}	10^4	10^4	10^6
0.02	10^{25}	10^{26}	10^{26}	10^{19}	10^{19}	10^{20}	10^{15}	10^{16}	10^{17}

and 1.4 Å cases, a decrease in the fusion rate with increased temperature may be expected. These predictions could be tested by externally increasing the temperature of an active fusion cell and by adjusting the deuteron concentration.

Thus, we see that an effective deuteron mass of only $\sim 10^{-1}$ times the free deuteron mass can account for the known positive experimental observations. The strong exponential dependence of the fusion rate on the concentration and effective mass is evident from Table II. This may in part explain the wide spread in positive and negative experimental results of different investigators.

Branching Ratio

The relevance of the Beuhler et al. experiments to cold fusion goes beyond the apparent need for a novel concept to explain the fusion rates. There are tantalizing indications in these experiments that the ratio between the neutron and proton branches of the $d-d$ reaction may be much smaller than unity. This is similar to the inference obtained in cold fusion from observation of neutron to triton yields in electrolytic cells.^{7,20}

As shown in Fig. 2 of Ref. 1, with an energy calibration of 8 keV/channel, the energy shifted ("to about 0.75 MeV") ^3He peak should appear at about channel 94 with an intensity equal to the triton peak around channel 110. However, even with a contribution from the tail of the gamma background, there is a distinct minimum in the energy spectrum in the vicinity of channel 94 showing that ^3He is not observed. Beuhler et al. with proper scientific caution allow that the ^3He ions "may be obscured by the x-ray background." Only if the energy calibration is significantly nonlinear can the ^3He be expected to fall in the gamma background (X-ray background). Furthermore, the area (integrated count) for the 1-MeV triton signal is roughly the same as the area of the 3-MeV proton signal. This would be expected in the absence of ^3He . For the normally expected approximately equal triton and ^3He branches, the area under the 1-MeV peak should be roughly double the area for the proton signal since the energy resolution is not sufficient to fully resolve the triton and ^3He peaks.

Consequently, their experiment provides evidence that the neutron to proton branching ratio is much smaller than unity. Iyengar²⁰ observed 10^{-8} for this ratio, a value also suggested by Wolf's measurements.⁷ If the branching ratio were close to unity, the neutron flux would be significantly above background and should not be difficult to measure. Nevertheless,

in each case the experimental evidence must of course be strengthened before far-reaching conclusions could be scientifically acceptable. We note that there may be valid theoretical arguments for a skewed branching ratio at low energies.¹² This technical note does not further address the issue of the $d-d$ branching ratio other than to stress that *it is of great scientific interest that the branching ratio be measured under cluster-impact conditions.*

Cold Fusion Conclusion

Equation (12) permits us to estimate how close two deuterons would have to approach at ambient temperature in a solid to account for reported fusion rates, neglecting effective mass or any other special effects. For a given deuteron proximity $2R$, the number density for cold fusion n_C can be greater than the number density for lukewarm fusion n_L , which at small R more than makes up for $E_C \ll E_L$. For small R , n_C represents a local rather than a global number density, just as n_L only represents the number density in the impact region. Care must be exercised in comparing the cold fusion rate where $n_C = (2R)^{-3}$ and the lukewarm rate where $n_L = (n_0)^{2/3}(2R)^{-1}$. Hence, a separation of $\sim 0.14 \text{ \AA}$ ($R = 0.07 \text{ \AA}$) would yield 0.1 W/cm^3 for cold fusion. Decreasing the separation to $\sim 0.12 \text{ \AA}$ ($R = 0.06 \text{ \AA}$) yields 20 W/cm^3 . Such close separations are not possible in equilibrium, where the closest separation is calculated to be 0.94 \AA (Refs. 8, 9, and 10), and would even be very difficult to achieve in non-equilibrium. Therefore, if fusion in the bulk is to account for this excess power generation, it appears that something extraordinary is a vital part of the tunneling process, such as a decreased effective mass of the fusing particles as proposed here. (Interestingly, Cohn and Rabinowitz have derived a classical analog to quantum tunneling.²⁵)

GENERAL CONCLUSIONS

This analysis clearly indicates that both lukewarm and cold fusion investigators report fusion rates that cannot be explained by conventional means. In addition, they both appear to favor the $t + p$ channel greatly over the $^3\text{He} + n$ channel. Even though it appears unlikely that a periodic structure is maintained in the cluster-impact experiments, it is interesting to note that as shown by Eq. (14) the effective mass scales $\propto (a^2E)^{-1}$. Hence, in the Beuhler et al. experiments a one-dimensional lattice compression of ~ 10 with $E \sim 100 \text{ eV}$ corresponds roughly to the same deuteron effective mass as the $E \sim 1 \text{ eV}$ cold fusion experiments.

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