

**CLUSTER-IMPACT FUSION:  
BRIDGE BETWEEN HOT AND COLD FUSION?\***

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**ABSTRACT**

Beams of  $D_2O$  clusters with 10 ~ 1000 eV per deuterium yield unexpectedly high fusion rates. This is an energy range intermediate between hot and cold fusion, and as such may serve as a link bridging the two regimes. We present a theoretical model capable of explaining apparently conflicting experimental results with beams of  $D_2O$  clusters in which fusion rates higher than expected were observed, and beams of  $D_2$  clusters in which no fusion was observed. Calculated results indicate that total deuterium-deuterium fusion rates can be enhanced by many orders of magnitude by the use of deuterium-heavy atom cluster beams. A set of experimental tests is proposed.

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## ABSTRACT

Beams of  $D_2O$  clusters with  $10 \sim 1000$  eV per deuterium yield unexpectedly high fusion rates. This is an energy range intermediate between hot and cold fusion, and as such may serve as a link bridging the two regimes. We present a theoretical model capable of explaining apparently conflicting experimental results with beams of  $D_2O$  clusters in which fusion rates higher than expected were observed, and beams of  $D_2$  clusters in which no fusion was observed. Calculated results indicate that total deuterium-deuterium fusion rates can be enhanced by many orders of magnitude by the use of deuterium-heavy atom cluster beams. A set of experimental tests is proposed.

## I. INTRODUCTION

Beuhler *et al.*<sup>1,2</sup> (BFF)<sup>1</sup> report cluster-impact fusion rates  $\sim 10^{25}$  times higher than expected<sup>3,4</sup> from single deuterium-deuterium ( $D-D$ ) collisions at center of mass (CM) energies of 150 eV/D, and  $\sim 10^{100}$  times higher at 15 eV/D. This is reminiscent of the unexpectedly high fusion rates reported for cold fusion.<sup>5,6</sup> The deuterium energy range for cluster-impact fusion is intermediate between hot and cold fusion, and as such may serve as a link bridging the two regimes. Several theoretical models<sup>7-9</sup> have been proposed to explain the discrepancy but fail to reproduce the observed fusion rates<sup>1</sup> by many orders of magnitude.<sup>10</sup> In a previous paper,<sup>3</sup> one of us demonstrated that the factor of  $10^{25}$  could not be accounted for by compression and electron screening alone which could at most account for only a factor of  $\sim 10^{10}$ . This left an unaccounted discrepancy of  $10^{15}$ .

Beuhler *et al.*<sup>1,2</sup> measured  $D-D$  fusion rates in a series of experiments in which singly charged clusters of  $D_2O$  molecules ( $(D_2O)_n^+$ ,  $n = 25 - 1300$ ) accelerated to 200 to 325 keV (with a beam current of  $\sim 1$  nA) were incident on  $TiD$ ,  $(C_2D_4)_n$  and  $ZrD_{1.65}$  targets. The unexpected results of these experiments with  $D_2O$  clusters which correspond to fusion rates  $\sim 10^{-1} s^{-1}/D-D$  appear to be contradicted by Fallavier *et al.*<sup>11</sup> (FKPRT) who carried out similar experiments using pure deuterium clusters ( $D_{200}^+ \sim D_{300}^+$ ) with kinetic energies of 100 - 150 keV in the same range of incident CM energy per deuteron (200 eV  $\sim$  375 eV)

and observed no  $D - D$  fusion events. Their upper limit<sup>11</sup> for the fusion rate is more than one order of magnitude below the observed value of BFF.<sup>1</sup>

In this paper, we describe in greater detail our recently proposed cluster-transport impact-fusion mechanism<sup>12</sup> to describe both these positive<sup>1,2</sup> and negative<sup>11</sup> results from the cluster-beam fusion experiments.<sup>1,2,11</sup> We show that heavy atoms such as  $O$  in the cluster, and  $Ti$ ,  $Zr$ , or  $C$  in the target are essential for obtaining high fusion rates by Rutherford double-backscattering and energy reflection enhancement of the  $D$ 's for the experiments as conducted, and thus we can reconcile the conflicting results. We also predict the conditions required for comparable yields from  $D$  and  $D_2O$  clusters. As previously suggested by one of us,<sup>4</sup> a resonance cross-section is proposed as an explanation for the behavior of the very low-energy data as the number of molecules in a cluster increases to  $n \approx 400 \sim 1000$ , or as the energy per  $D$  decreases to 15 eV in the CM system.

## II. OUR THEORETICAL MODEL AND ANALYSIS

In our model, molecular clusters  $(D_\alpha X_\beta)_n^+$ , of  $\sim 10 \text{ \AA}$  size disintegrate into  $(\alpha + \beta)n$  atoms upon impact with the target during  $\Delta t \sim 10^{-14} s$  for a cluster velocity of  $\sim 10^7 cm/s$ . During  $\Delta t$ , we assume that a local Maxwell-Boltzmann (M-B) distribution  $f(\vec{r}, \vec{v}, t)$  is developed for the  $(\alpha + \beta)n$  atoms due to a local thermalization using a fraction of the total kinetic energy  $E_t$ . Although the total number of atoms per cluster is limited to  $(\alpha + \beta)n$ , the total number  $N_t$  of atoms involved with a cluster-beam with an intensity of  $\sim 1 nA$  is large,  $N_t = 0.625 \times 10^{10}(\alpha + \beta)n/s$  ( $\approx 2 \times 10^{12}/s$  for  $(D_2O)_{100}$ ), and hence can be regarded as a reasonable statistical system moving with the total kinetic energy  $E_t$ . Recent molecular-dynamics simulations by both Carraro *et al.*<sup>7</sup> and Shapiro and Tombrello<sup>13</sup> indicate that the high energy tail of the energy distribution decays approximately exponentially like the M-B distribution as produced in the early stages of the collisional cascade initiated by cluster impacts.

### II.A. Energy Enhancement by Rutherford Double-Backscattering

The enhancement of the  $D - D$  fusion rate over the conventional estimate is expected for the case of the  $(D_2O)_n^+$  beam<sup>1</sup> but not for the case of the  $(D)_n^+$  beam<sup>11</sup> because deuteron thermalization mainly occurs due to multiple scatterings between incident deuterons, heavy target atoms (e.g.,  $Ti$ ), and oxygen atoms in the clusters after the breakup of the  $D_2O$  atoms. For example, if an incident deuteron with an initial (cluster) velocity  $v_i$  in the leading edge of the cluster backscatters first from a target titanium atom and then again from an incoming oxygen atom, the deuteron can be accelerated to a final maximum velocity of  $v_f = [((M_{Ti} - m)/(M_{Ti} + m))((M_o - m)/(M_o + m)) + 2M_o/(M_o + m)]v_i \approx 2.49 v_i$ ; i.e., the kinetic energy of the incident deuteron can increase by a factor of  $(v_f/v_i)^2 \approx 6.2$ . In contrast, for the case of a pure deuterium  $(D)_n^+$

beam, similar double backscatterings of an incident deuteron from a target titanium atom and then from another incoming deuterium atom cannot increase the deuteron kinetic energy since the final maximum deuteron velocity is given by  $v_f = [((M_{Ti} - m)/(M_{Ti} + m))((m - m)/(m + m)) + 2m/(m + m)]v_i \approx v_i$ . Therefore, the presence of heavy atoms in the  $(D_2O)_n^+$  cluster efficiently populates the high energy tail of the deuteron M-B distribution via multiple collisions that lead to substantial thermalization during the impact time period of  $\Delta t \approx 10^{-14}$  seconds, and hence to enhanced reaction rates. The pure deuterium  $(D)_n^+$  cluster beam will not achieve as high a degree of thermalization as the  $(D_2O)_n^+$  clusters upon impact, and only inefficiently populates the high energy tail because of the absence of heavy atoms in the  $(D)_n^+$  cluster; this means less enhanced (smaller) reaction rates.

One can now ask whether Rutherford double-backscattering can actually have a significant effect on the high-energy tail of the deuteron velocity distribution. We consider the specific case of the original BFF experiment. We define the fraction of deuterons which undergo backscattering as

$$F = Pt = \tilde{n}\sigma vt \quad (1)$$

where  $P = \tilde{n}\sigma v$  is the probability per unit time for a single deuteron to backscatter from a layer of target atoms (titanium),  $\tilde{n}$  is the density of the target at its surface layer,  $v$  is the incoming deuteron velocity, and  $\sigma$  is the backscattering cross-section

$$\sigma = \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} d\theta \sigma(\theta) \sin \theta \quad (2)$$

Here we take  $\sigma(\theta)$  to be the Coulomb differential scattering cross-section:

$$\sigma(\theta) = \left( \frac{Z_d Z_t e^2}{2\tilde{\mu}v^2} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} \quad (3)$$

where  $Z_d, Z_t$  are the atomic numbers for the deuteron and target atoms, respectively, and  $\tilde{\mu}$  is the reduced mass. For the present case,  $\tilde{\mu} \approx m_d$  is the deuteron mass. After inserting (3) into (2) and integrating, we find that

$$\sigma = \pi \left( \frac{Z_d Z_t e^2}{m_d v^2} \right)^2 = 1.63 \times 10^{-20} \left( \frac{Z_d Z_t}{E_d(\text{keV})} \right)^2 \text{cm}^2 \quad (4)$$

where  $E_d = m_d v^2/2$ . In eq. (1),  $t$  is the time it takes a deuteron to penetrate the target layer, assuming a hard-sphere interaction. Therefore if one defines  $d$  to be approximately the target atomic or ionic diameter, then  $t \approx d/v$  so that eq. (1) thus gives an approximate backscattering probability:

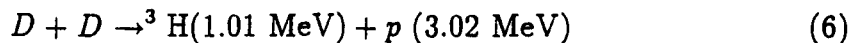
$$F = \tilde{n}\sigma d = 1.63 \times 10^{-20} \text{cm}^2 \tilde{n} d \left( \frac{Z_d Z_t}{E_d(\text{keV})} \right)^2 \quad (5)$$

(for the maximum energy enhancements which we calculate below for double-backscattering,  $F$  calculated from eq. (5) is an overestimate, since maximum energy enhancement occurs when the scattering angle is  $\theta \approx 180^\circ$ —eq. (5) describes the *total* backscattering probability of the deuteron anywhere into the backward hemisphere for any final deuteron kinetic energy). For the case of a 0.1 keV deuteron backscattering off a titanium lattice surface, we have  $\bar{n} = n_{Ti} \approx 5.7 \times 10^{22} \text{ cm}^{-3}$  and  $d \approx 2 \times 10^{-8} \text{ cm}$  ( $\sim$  atomic diameter), so that  $F = F_{DTi} \approx 0.90$ . For the case of a  $\sim 0.1$  keV deuteron backscattering from an oxygen atom in the  $D_2O$  cluster,  $\bar{n} = n_{D_2O} \approx 3.0 \times 10^{22} \text{ cm}^{-3}$  and  $d \approx 3 \times 10^{-8} \text{ cm}$ , so that  $F = F_{DO} \approx 0.094$ . Therefore, taking into account that a deuteron which backscatters from titanium loses very little energy, the probability that a 0.1 keV deuteron first backscatters from a titanium nucleus and then off an oxygen nucleus is  $F_1 = F_{DTi} F_{DO} = (0.90)(0.094) = 0.085$ , i.e., a 100 eV cluster deuteron has a 1 in 12 chance of undergoing a Rutherford double-backscatter and being accelerated to a kinetic energy as large as 620 eV. Note that  $F_1$  is probably conservative, since the deuteron interacts electro-dynamically with the heavy atoms well beyond the hard-sphere radius.

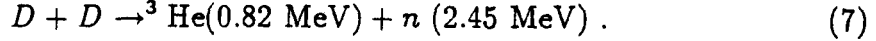
Observe that eq. (5) indicates that  $F \propto E_d^{-2}$ , so that if a 620 eV deuteron undergoes the double backscatter described above, it has a probability of  $F_1 = 0.085/(6.2)^4 = 5.7 \times 10^{-5}$  of doing so. Therefore the probability that a 100 eV deuteron will undergo two double backscatterings will be  $F_2 = F_1(.1 \text{ keV}) \times F_1(.62 \text{ keV}) = 4.9 \times 10^{-6}$ , if the deuteron picks up maximum energy after it backscatters from an oxygen for the first time. The above value for  $F_2$  is almost certainly a lower limit, given that the deuteron will generally not pick up the maximum energy available from the oxygen atoms, and given the conservative nature of  $F_1$ . Therefore,  $F_2$  can easily be a few orders of magnitude larger, and we may conclude that for low energy ( $\sim 100$  eV) deuterons single double-backscattering most likely (and double double-backscattering possibly) contributes significantly to the thermalization process in cluster breakup. Note that  $F_1$  and  $F_2$  will be significantly higher for smaller  $E_d$  than for larger  $E_d$  (since  $F_2 \propto E_d^{-4}$ ), so that large clusters with small  $E_d$  will be affected much more by this multi-scattering process than small clusters with large  $E_d$ . Therefore, this process may make a contribution to the slow drop off of yield with increase in cluster size as observed by BFF<sup>1</sup> (see below). It is also possible that for small  $E_d$ , atomic backscattering may have a larger cross-section than the higher-energy Rutherford backscattering, thus increasing the probability for energy enhancement by multiple backscattering at lower energies.

## II.B. Low-Energy $D - D$ Fusion Cross Sections

For  $D - D$  fusion, the two dominant channels are



and



Experimental values of the cross-section,  $\sigma(E)$ , for (6) and (7) can be parameterized as<sup>14</sup>

$$\sigma(E) = \frac{S(E)}{E} e^{-(E_G/E)^{1/2}} \quad (8)$$

where  $E_G$  is the "Gamow energy" given by  $E_G = (2\pi\alpha Z_p Z_D)^2 \mu c^2 / 2$  or  $E_G^{1/2} = 31.39 (\text{keV})^{1/2}$  for the deuteron reduced mass  $\mu \approx m/2$ .  $E$  is in units of keV in the CM reference frame. The astrophysical factor,  $S(E)$ , is extracted from the experimentally measured values<sup>14</sup> of the cross-section,  $\sigma(E)$ , for  $E \gtrsim 4$  keV and is nearly constant<sup>14</sup> ( $S(E) \approx 52.9 \text{ keV} - b$ ) for both reactions (6) and (7) in the energy range of interest here,  $E \lesssim 1$  keV, assuming an equal branching ratio. However, if  $\sigma(E)$  happens to have resonant behavior at low energies as previously suggested by one of us,<sup>4</sup> the extrapolation method may yield erroneous values for  $\sigma(E)$  or  $S(E)$  at low energies, since the non-resonant relation, eq. (8), is not applicable to cases involving resonant reactions.<sup>4,14</sup>

For the case of possible low-energy resonances,<sup>4</sup> the cross-section for reactions (6) or (7) is given by (( $\alpha$ ) = (6) or (7))

$$\sigma^{(\alpha)}(E) = \sigma(E) + \sigma_R^{(\alpha)}(E) + \text{interference terms} \quad (9)$$

where the resonance cross-section is parameterized as<sup>14</sup>

$$\sigma_R^{(\alpha)}(E) = \frac{\pi \hbar^2}{M_D E} \frac{\omega^{(\alpha)} \Gamma_{\text{ent}} \Gamma^{(\alpha)}}{(E - E_r^{(\alpha)})^2 + \Gamma^2 / 4} \quad (10)$$

where  $\omega^{(\alpha)}$  is a spin statistics factor,  $\Gamma_{\text{ent}}$  and  $\Gamma^{(\alpha)}$  are partial widths for the entrance and exit ( $\alpha$ ) channels, respectively, and  $\Gamma$  is the total width.

## II.C. Incomplete Thermalization with a Shifted Maxwell-Boltzmann Distribution

As a lowest-order solution to the Boltzmann transport equation, the local M-B distribution for deuterons in clusters can be written as<sup>15</sup>

$$f(\vec{r}, \vec{v}, t) = f(\vec{v}) = \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-m(\vec{v} - \vec{v}_d)^2 / 2kT} \quad (11)$$

where  $m$  is the deuteron rest mass and  $\vec{v}_d$  is the average deuteron transport velocity. Both  $\vec{v}_d$  and  $T$  are functions of  $\vec{r}$  and  $t$ . Before the cluster impacts on the target,  $\vec{v}_d(\vec{r}, t)$  has a constant value  $\vec{v}_0$  which is related to  $E_t$  by  $m v_0^2 / 2 = E_t m / (\alpha m + \beta M) n$  with rest mass  $M$  for the atom  $X$ .  $f(\vec{v})$  as given by eq. (11)

has the following properties:  $\langle \vec{v} \rangle = \int \vec{v} f(\vec{v}) d^3v = \vec{v}_d$ ,  $\langle m(\vec{v} - \vec{v}_d)^2/2 \rangle = 3kT/2$ , and  $f(\vec{v}) \rightarrow \delta(\vec{v} - \vec{v}_0)$  in the limit as  $kT \rightarrow 0$ , corresponding to a monoenergetic deuteron beam with deuteron kinetic energy  $mv_0^2/2$  (no thermalization after impact). We note that the deuteron transport kinetic energy  $mv_d^2/2$  during the impact time ( $\Delta t \sim 10^{-14}$ s) is restricted by  $0 \leq mv_d^2/2 \leq mv_0^2/2$ . The average kinetic energy  $\langle mv^2/2 \rangle$  for a deuteron due to impact thermalization is now given by

$$\langle mv^2/2 \rangle = (m/2) \int v^2 f(\vec{v}) d^3v = mv_d^2/2 + 3kT/2. \quad (12)$$

The thermalization energy  $3kT/2$  comes from energy transferred from the heavy atoms of mass  $M \gg m$  by multiple collisions between the deuterons and the heavier atoms. For complete thermalization,  $v_d = 0$  in eq. (11) (similarly,  $v_X = 0$  in an equation like eq. (11)) so that  $3kT/2 = 3kT_{max}/2 = E_t/(\alpha + \beta)n$  (and  $kT = kT_{max}$  also for the  $X$  atoms) using the equipartition theorem for thermal equilibrium. Total energy conservation for each cluster requires that  $mv_d^2/2 + 3kT/2 = 3kT_{max}/2$  and  $Mv_X^2/2 + 3kT_X/2 = 3kT_{max}/2$  with the restrictions that  $mv_d^2/2 \leq mv_0^2/2$  and  $Mv_X^2/2 < Mv_0^2/2$ . At thermal equilibrium,  $kT_X = kT = kT_{max}$  ( $v_d = v_X = 0$ ), while for the nonequilibrium situation,  $kT_X \neq kT$  ( $v_d \neq 0$ ,  $v_X \neq 0$ ). Although the M-B distribution with  $\vec{v}_d = 0$  in eq. (11) has been used in previous theoretical studies,<sup>7-10</sup> the  $\vec{v}_d \neq 0$  case for cluster-impact fusion was first studied by us.<sup>12</sup>

We assume that the deuteron flux described by  $f(\vec{v})$ , eq. (11), after impact thermalization, impinges on the target (TiD, etc.). For the local M-B distribution,  $f(\vec{v})$ , eq. (11), the fusion rate  $R_{calc}$  for the reaction  $D(D, p)T$  is given by

$$R_{calc} = (\Phi/v_0) \int v f(\vec{v}) P(v) d^3v \quad (13)$$

where  $\Phi = n_i A v_0$  (number of incident  $D$ 's per unit time) is the initial  $D$  flux incident on the target area  $A$  with an incident deuteron density  $n_i$ , and  $P(v)$  is the probability for a deuteron to undergo a fusion reaction while slowing down in the target, which can be written as  $P(v) = n_D \int_0^{E_i} dE_D \sigma(E_{DD})/|dE_D/dx|$  with  $E_i = mv^2/2$ .  $E_D$  and  $E_{DD}$  are the deuteron kinetic energies in the laboratory (LAB) and CM frames, respectively ( $E_{DD} = E_D/2$ ),  $dE_D/dx$  is the stopping power<sup>15</sup> and  $\sigma(E)$  is the  $D - D$  fusion cross-section.

For order of magnitude estimates, the  $D - D$  fusion reaction rate,  $R_{calc}$ , eq. (13), can be approximated as

$$R_{calc} \approx g(\Phi/v_0) \Delta x \Lambda(E_d) = g(0.261 \times 10^{-12} \text{ s}) \Phi \Lambda(E_d) \quad (14)$$

where  $\Delta x$  is an effective interaction thickness ( $\sim 10^2 - 10^3 \text{ \AA}$ ) of the target given by  $\Delta x = \int_0^{E_0} |dE_D/dx|^{-1} dE_D \approx 0.81 \times 10^{-5} (E_0(\text{in keV}))^{1/2} \text{ cm}$  with  $E_0 = mv_0^2/2$ , and  $\Lambda(E_d)$  is given by



$$\Lambda(E_d) = n_D \langle \sigma v \rangle = n_D \int \sigma(v) v f(\vec{v}) d^3v, \quad (15)$$

for a deuteron density  $n_D$  in the target and  $E_d = mv_d^2/2$ .  $n_D$  is assumed to be the same as the titanium density  $n_{Ti} = n_D = 5.7 \times 10^{22} \text{ cm}^{-3}$ .

Following initial contact with the target, the cluster is no longer a closed system and a small fraction of the incident atoms is lost out of the interaction region mostly in the backward hemisphere. The factor  $g$  in eq. (14) is included to account for the fraction of incident  $D$ 's that are lost due to backscattering. In the calculations,  $g$  is set to 0.5, which is appropriate for the case of a completely thermalized cluster. For incomplete thermalization,  $g$  should be between 0.5 and 1.0, but, as a conservative estimate, it is kept at 0.5.

With  $f(\vec{v})$  given by eq. (11),  $\Lambda(E_d)$ , eq. (15), becomes

$$\begin{aligned} \Lambda(E_d, kT) &= n_D \frac{2}{v_d} \left( \frac{2}{\pi m kT} \right)^{1/2} \int_0^\infty dv e^{-2(E_d/m)^{1/2}/v} S(mv^2/4) \\ &\times e^{-m(v^2+v_d^2)/2kT} \left( e^{mvv_d/kT} - e^{-mvv_d/kT} \right). \end{aligned} \quad (16)$$

For the case of  $v_d = 0$  corresponding to a completely thermalized M-B distribution, eq. (15) or eq. (16) reduces to<sup>17</sup>

$$\tilde{\Lambda}(E_T) = \Lambda(0, kT) = n_D \left( \frac{8}{m\pi} \right)^{1/2} \frac{1}{(E_T)^{3/2}} \int_0^\infty \sigma(E/2) e^{-E/E_T} E dE \quad (17)$$

with  $E_T = kT$  for  $E$  in the LAB frame, and

$$\tilde{\Lambda}(E_T) = \Lambda(0, kT) = n_D \left( \frac{8}{\mu\pi} \right)^{1/2} \frac{1}{(E_T^{cm})^{3/2}} \int_0^\infty \sigma(E_{cm}) e^{-E_{cm}/E_T^{cm}} E_{cm} dE_{cm} \quad (18)$$

with  $E_{cm}(= E/2)$  in the CM frame, the reduced mass,  $\mu = m/2$  and  $E_T^{cm} = kT/2$ . In one double backscattering of a  $D$  from a target  $T_i$  and then from a projectile  $O$ , the  $D$  increases its energy by a maximum factor of 6.2. For two double backscatterings the maximum energy gain factor is  $(6.2)^2 = 38.4$ . Thus Rutherford double-backscattering can quickly populate the high-energy tail of the M-B distribution during complete or partial thermalization of the  $(D_2O)_n^+$  cluster. Hence, the upper limit of integration in eqs. (16) and (17) is expected to be greater than approximately  $500 \text{ eV} \times 38.4 \approx 19 \text{ keV}$ . We therefore set the upper limit to be infinity in our calculations; this is expected to give our results an overestimate of one order of magnitude or less.

Note that for monoenergetic deuterons,  $f(\vec{v}) = \delta(\vec{v} - \vec{v}_0)$ , and  $R_{calc}$  as given by eq. (13) reduces to<sup>18</sup>

$$R_{calc}(mono) = \Phi P(E_0) = \Phi(2.3 \times 10^{-6}) \exp(-44.40/\sqrt{E_0(\text{keV})}) . \quad (19)$$

## II.D. Cluster-Size Distribution

As part of the experiments of Beuhler *et al.*<sup>2</sup>, the incoming cluster beam before acceleration was mass-analyzed, and found to have a shape that is well-described by a Gaussian or Poisson distribution.<sup>5</sup> The distribution was found to typically have a full-width half-maximum  $\Delta n \approx 0.38$ ; the distribution peak,  $\bar{n}$ , is the value ascribed to the cluster size (i.e.,  $(D_2O)_n^\pm$ ) for a given run of the experiment.

If we represent the cluster size distribution as Gaussian in the experiments of Beuhler *et al.*, and assume complete thermalization ( $v_d = 0$  in eq. (11)), we may define the following cluster size ( $n$ ) dependent M-B-like velocity distribution for use in our calculation:

$$\begin{aligned} f(v, n) &= \bar{N} \exp\left(\frac{-mv^2}{2kT}\right) \exp\left(\frac{-(n - \bar{n})^2}{2\bar{\sigma}^2}\right) \\ &= \bar{N} \exp\left(\frac{-9nE_{cm}}{E_t}\right) \exp\left(\frac{-(n - \bar{n})^2}{2\bar{\sigma}^2}\right) , \end{aligned} \quad (20)$$

since  $E_{cm} = mv^2/4 = \mu v^2/2$  and  $kT = 2E_t/9n$ , where

$$\bar{\sigma} = \frac{(\Delta n)\bar{n}}{2\sqrt{\ln 4}} = \frac{0.19\bar{n}}{\sqrt{\ln 4}}$$

is the Gaussian width of the  $(D_2O)_n^\pm$  mass distribution. We solve for the normalization factor,  $\bar{N}$ , via the requirement that  $1 = \int d^3v \int dn f(v, n)$ , and then solve for the fusion rate

$$\Lambda(\bar{n}) = n_D \langle \sigma v \rangle = n_D \int d^3v \int dn \sigma v f(v, n) . \quad (21)$$

Using eq. (8) for  $\sigma$ , we then find

$$\bar{N} = \frac{\mu^{3/2}}{4\pi\sqrt{2}} \left[ \int_0^{E_c} \sqrt{E} I(E) dE \right]^{-1} , \quad (22)$$

and

$$\Lambda(\bar{n}) = \sqrt{\frac{2}{\mu}} \left[ \int_0^{E_c} \sqrt{E} I(E) dE \right]^{-1} \int_0^{E_c} S(E) e^{-(E\sigma/E)^{1/2}} I(E) dE , \quad (23)$$

where

$$\begin{aligned}
I(E) &= \int_{\hat{n}}^{\infty} e^{-9n} E/E_t e^{-(n-\bar{n})^2/2\bar{\sigma}^2} dn \\
&= \frac{1}{2} \sqrt{2\pi\bar{\sigma}} \exp\left(\frac{E}{E_T^{cm}} - \frac{(\Delta n)^2}{16 \ln 2} \left(\frac{E}{E_T^{cm}}\right)^2\right) \\
&\quad \times \operatorname{erfc}\left(\frac{\Delta n}{4\sqrt{\ln 2}} \frac{E}{E_T^{cm}} + \frac{(\hat{n} - \bar{n})}{\sqrt{2}\bar{\sigma}}\right).
\end{aligned} \tag{24}$$

Here  $E_c$ ,  $\hat{n}$ , and  $\operatorname{erfc}(x)$  are the cutoff energy for the velocity portion of the distribution, the cutoff cluster size for the mass portion of the distribution, and the complimentary error function, respectively. In Eqs. (22) - (24),  $E = E_{cm} = mv^2/2$ . Note that one can have  $\hat{n} \geq 1$ ; it is claimed by Beuhler et al.<sup>2</sup> that  $\hat{n} = \bar{n}/2$ .

### III. COMPARISON WITH PREVIOUS THEORETICAL WORK

In this section, we discuss some of the previous theoretical work by others in order to clarify differences between their models and ours, to describe the relevance of the previous work to our work, and to indicate some obvious errors and inconsistencies in the previous work.

#### III.A. Critique of EMR's Work

In a recent theoretical paper,<sup>8</sup> Echenique *et al.* (EMR) claim to provide an explanation for the recent cluster-impact fusion experiments.<sup>1</sup> However, both papers<sup>1,8</sup> contain several errors.

First, both Refs. 1 and 8 address resolving a discrepancy of only  $10^{10}$  between measured and expected reaction rates (assuming a monoenergetic deuterium velocity distribution). However, more correctly, a difference of  $10^{25}$  must be explained.<sup>3,4</sup> Our calculation<sup>3,4</sup> shows a discrepancy of  $10^{25}$  rather than the one of  $10^{10}$  suggested by BFF<sup>1</sup> and addressed by EMR.<sup>8</sup> The difference between  $10^{25}$  and  $10^{10}$  is the difference between properly doing the calculation in the CM frame, and doing it improperly in the LAB frame. Their error<sup>1,8</sup> is due to the incorrect use of  $E(LAB)$  (instead of using the correct  $E(CM) = E(LAB)/2$ ) in  $\sigma(E)$  (eq. (8)) in their paper,<sup>1,8</sup> as first pointed out by one of us.<sup>3</sup> References 14 and 17 also provide clarification on this point.

Second, eq. (3) of Ref. 8 (compare with our eq. (18)) would be correct for CM energies if  $\sqrt{2}$  is eliminated and their  $E_0 = kT/2$  assuming a M-B deuterium velocity distribution. However, a crucial topic is the determination of  $E_0$  given  $N$  particles obeying a M-B velocity distribution with total LAB cluster kinetic energy  $E_t$ . A simple calculation shows that  $E_t = N(3 kT/2) = 3NE_0$ , or  $E_0 = E_t/3N$ . For their example given below eq. (3) in Ref. 8 where  $E_t = 300$  keV and  $N \sim 1000$ ,  $E_0$  is given as 500 eV instead of the correct value of 100 eV. When the correct

value of 100 eV is used to calculate the fusion rate, a value of  $R \approx 10^{-8} s^{-1}$  is obtained instead of the quoted  $0.08 s^{-1}$  for the incorrect  $E_0 = 500$  eV.

The choice of  $E_0$  cannot be arbitrarily independent of their choice of distribution (equilibrium, non-equilibrium, etc.). The choice of  $E_0$  is constrained by both the conservation of energy and the conservation of the number of particles. Their eq. (3) incorporates a M-B distribution as they state.<sup>8</sup> They did an equilibrium calculation. The words "non-equilibrium" do not occur anywhere in their paper,<sup>8</sup> though they do suggest future calculations that are conceptually of such a nature. We have carried out a non-equilibrium calculation<sup>12</sup> as described earlier in this paper. We find a significantly lower  $E_0$  than EMR's 500 eV rather than a higher  $E_0$  as claimed by EMR. Our paper shows that a shifted local M-B distribution as a solution to the Boltzmann transport equation gives a lower fusion rate than a fully thermalized distribution precisely because there are fewer particles in the high energy tail. We emphasize that non-equilibrium implies a smaller  $E_0$  than the 500 eV used by EMR,<sup>8</sup> and that this choice by EMR violates conservation of energy and number of particles. Furthermore, EMR never explicitly give  $E_0$  in terms of total cluster energy and cluster size in their paper,<sup>8</sup> thereby rendering Fig. 1 of EMR unverifiable.

Third, the theoretical curve in Fig. 1 of Ref. 8 cannot be reproduced without more information on the dependence of the number of particles in the impact region ( $N$ ) as a function of the number of  $D_2O$  molecules in the cluster ( $n$ ) and total cluster energy ( $E_t$ ). With a simple model where the cluster thermalizes upon impact before equipartitioning its energy with the target, one obtains  $E_0 = E_t/9n$ , and for the example mentioned above with  $n = 100$ , the result of  $E_0 = 333$  eV yields  $R \approx 0.003 s^{-1}$ , which is about 25 times smaller than the value of  $0.08 s^{-1}$  calculated with the physically unacceptable value of  $E_0 = 500$  eV.

Furthermore, Fig. 1 in Ref. 8 (reproduced as our Figure 1) is misleading if not examined carefully. Their calculated<sup>8</sup> and experimental<sup>1,8</sup> values were normalized to unity at  $E_t = 300$  keV, which gives rise to the potentially deceptive conclusion that their calculated rates fit the experimental values. However, for our simple model where  $E_0 = E_t/9n$  and a comparison calculation where  $E_0 = E_t/6n$ , the normalized values differ only slightly from the plotted curve in Fig. 1 of Ref. 8. But, when the actual rates are compared, we find that our calculated values differ from the experimental results by some four orders of magnitude for  $E_0 = E_t/9n$  and by two orders of magnitude for  $E_0 = E_t/6n$ , while it is unclear from Ref. 8 what their actual reaction rates are for the case of  $n = 150$  (Fig. 1) since they give neither the details of their model nor a calculated value.

Finally, calculations done with a M-B distribution and  $E_0 = E_t/9n$  give reaction rates that differ from those in Fig. 3 of BFF<sup>1</sup> (for  $E_t = 300$  keV) by 10 orders of magnitude for  $n \approx 1000$ . This appears to contradict the statement by EMR<sup>8</sup> that their model qualitatively explains the dependence of the fusion rate  $R$  on cluster size.

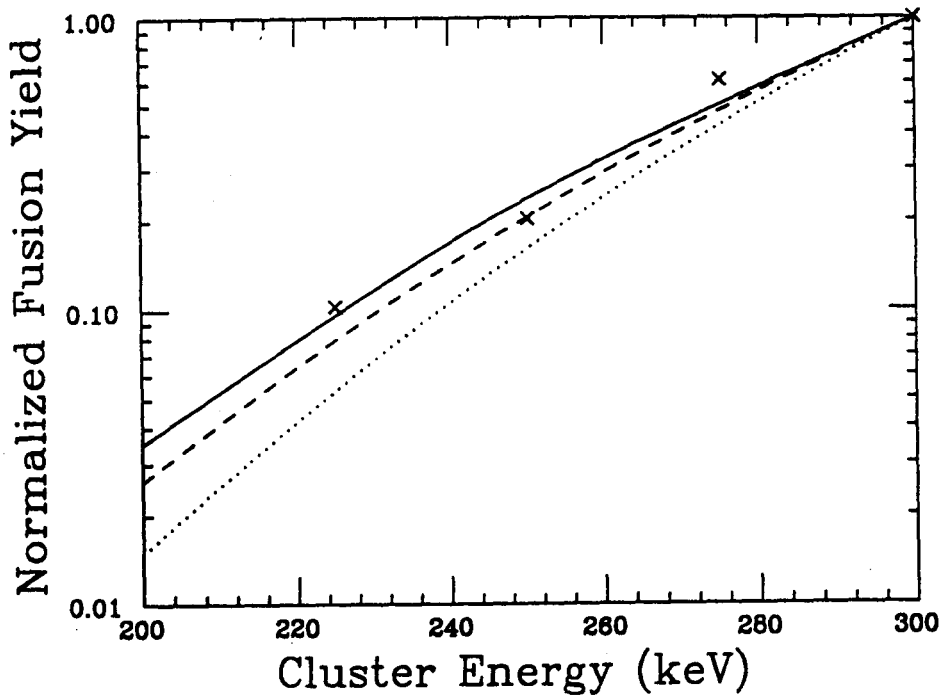


Figure 1: Normalized deuteron-deuteron fusion rate  $R$  for  $(D_2O)_{150}$  clusters incident on a  $TiD$  target as a function of total cluster laboratory energy  $E_t$ . The experimental points ( $\times$ ) and the fit of Echenique et al. (solid line) were extracted from Fig. 1 of Ref. 8. The other two curves are from our fit from eq. (3) of Ref. 8 (i.e., our eq. (18) with  $E_T^{\sigma n}$  replaced by  $E_o$ ) for  $E_o = E_t/6n$  (dashed line) and for  $E_o = E_t/9n$  (dotted line) for  $n = 150$ .

### III.B. Molecular Dynamics Simulations

Molecular dynamics (MD) simulations carried out by Carraro *et al.*<sup>7</sup> and also by Shapiro and Tombrello<sup>13</sup> show that the density in the interaction region reaches its maximum value of about twice the initial target density during the first  $\sim 10^{-14}s$ . From the above result of MD simulations, it is tempting to conclude that during this  $10^{-14}s$  energy not only is being shared among the atoms of the cluster but among a similar number of target atoms. However, such a conclusion may be premature since it is based on the assumption of the equipartition of energy among both target and incident cluster atoms, which is not applicable to incomplete thermalization.

The increase of the target density during the first  $10^{-14}s$  of MD simulations does not necessarily imply equipartition of energy among both target atoms and incident cluster atoms, since they are in a non-equilibrium situation. The incident deuterium atoms will carry most of the available energy due to efficient energy enhancement mechanisms such as the double backscattering mechanism described

above, thus contributing to the high velocity components described by the non-equilibrium velocity distribution,  $f(\vec{v})$  of eq. (11), while the target deuterium (and  $Ti$  and  $O$ ) atoms in the impact region will share a substantially smaller fraction of the available energy due to the fact that there are no efficient energy transfer mechanisms for them to populate higher velocity components during the first  $\sim 10^{-14}$ s. Therefore, the effective temperature for the incident projectile deuterium atoms can be substantially higher than that of the target deuterium (and  $Ti$  and  $O$ ) atoms and also higher than if the energy were shared equally among both target and projectile atoms, because not all the target atoms in the impact region can share in the equipartition of energy. For our shifted M-B distribution (when  $v_d = v_0$ ), the effective temperature is 70% of the maximum thermalization temperature. As can be seen for incident  $D_2O$  from our Table I, column 6, the fusion rate is decreased very little from the complete thermalization rate, at most by a factor of four for the  $n = 75$  case.

A compressional increase in projectile density,  $n_D^{proj}$ , and target density,  $n_D^{targ}$ , by a factor of two can increase the fusion rate by more than an order of magnitude. The increased density contributes in two ways. The linear dependence of the fusion rate per  $D - D$  collision is a minor contributor. The major contribution comes from the greatly increased probability of double backscattering which is proportional to  $(n_D^{proj} n_D^{targ})^b$ , where  $b$  is the number of double backscatterings (see Sect. II.A). For a density increase of two, with two double-backscatterings, the double-backscattering probability is increased by a factor of  $(2 \times 2)^2 = 16$ . This means a  $16 \times 2 = 32$  times increase in the fusion rate from both contributions combined. In presenting a conservative calculation, we have not yet put in the effect of increased density and have only used the uncompressed densities for our results presented here.

Carraro *et al.*<sup>7</sup> carried out MD simulations of the impact of clusters composed of  $n = 32, 54, 80, 128,$  and  $180$  heavy-water molecules at  $E_t = 100$  keV on a target composed of 1024 heavy-water molecules in a cubic box of  $32 \text{ \AA}$  on a side ( $\sim 10$  atomic layers). Their result with a few cluster impacts for the energy spectrum of target deuterons for  $n = 128$  at time  $t = 4 \times 10^{-14}$ s shows a spectrum approximately proportional to  $E^{-1.6}$  with a maximum energy cutoff at  $\sim 300$  eV ( $\sim 4$  times the incident deuteron energy of  $E_0 = 100 \text{ keV}/(10(128)) \approx 78$  eV).

Although their results<sup>7</sup> with an energy cutoff at  $\sim 300$  eV with only a few cluster impacts is consistent with that of a single knock-on process ( $78 \text{ eV} \times 3.16 \approx 247$  eV), their MD results<sup>7</sup> for the energy spectrum are not applicable to our non-equilibrium velocity distribution  $f(\vec{v})$ , eq. (11), for the projectile deuterons, since their spectrum is for the target deuterons. Their results for the target deuteron energy spectrum would be relevant to the case of  $(H_2O)_{115}$  cluster impact on a  $(C_2D_4)_n$  target for which the  $D - D$  fusion rate was observed by Beuhler *et al.*<sup>2</sup> to be about 5% of that for  $(D_2O)_n$  clusters on the same target. Although Shapiro and Tombrello<sup>13</sup> performed their MD simulations with several thousand cluster impacts, their results are not applicable to the case of  $D_2O$  cluster im-

pact, since they simulate the  $(Al)_n - Au$  and  $(Al)_n - Al$  cases for which double backscattering would be inefficient for energy enhancement and other effective energy-enhancement mechanisms are not available because  $(Al)_n$  (like  $(D)_n^+$ ) does not contain heavier atomic partners as in the case of  $(D_2O)_n^+$  incident on  $TiD$  or  $(C_2D_4)_n$ . In fact, the results of their MD simulations<sup>13</sup> show that the energy of an individual atom can only increase by a maximum factor of two over the incident energy per atom, with the probability for such events decreasing very rapidly with increasing energy. Their results<sup>13</sup> are relevant to and consistent with our description and results for  $(D)_n^+$  (but not for  $((D_2O)_n^+)$  cluster impact on  $TiD$  or  $(C_2D_4)_n$  targets in the context of the cluster-transport impact-fusion mechanism.

There is an error in Carraro et al.<sup>7</sup> Their eq. (10) is incorrect. The text on page 1382 reads, "In the case where a deuteron of energy  $U$  backscatters from a titanium atom and then again from the oxygen, the final energy  $\tilde{U}$  of the deuteron is  $\tilde{U}/U = [((M_0 - M_D)/(M_0 + M_D))(2M_{Ti}/(M_{Ti} + M_D))]^2 \approx 2.23$ ."<sup>7</sup> This equation with  $m = M_D$  should be  $\tilde{U}/U = [1 + ((M_0 - m)/(M_0 + m))((M_{Ti} - m)/(M_{Ti} + m))]^2 \approx 2.94$  if we take  $U$  to be the maximum knock-on energy a target deuteron can get from a projectile oxygen. That is,  $U = mv_d^2/2$ , with  $v_d = (2M_0/(M_0 + m))V_0$ , where  $V_0$  is the velocity of an incident oxygen and  $v_d$  is the velocity of a (knocked-on) target deuteron.  $U$  is arbitrary in Ref. 7. In general, this ratio is  $[(2M_0/(M_0 + m))(V_0/v_d) + ((M_0 - m)/(M_0 + m))((M_{Ti} - m)/(M_{Ti} + m))]^2$ . Since  $V_0/v_d$  can be arbitrarily large, one can have  $\tilde{U}/U \gg 2.94$ . This error propagates into their<sup>7</sup> eq. (11), etc..

The ratio of the final energy of a projectile deuteron,  $E_{pf}$ , to its initial energy can also be much larger. As discussed previously, if a projectile deuteron of initial energy  $E_{pi}$  backscatters elastically from a titanium and then again from an incoming oxygen, then by conservation of momentum and kinetic energy,  $E_{pf}/E_{pi} = [((M_{Ti} - m)/(M_{Ti} + m))((M_0 - m)/(M_0 + m)) + 2M_0/(M_0 + m)]^2 = 6.2$ . The total energy gain,  $E_{tf}$ , from this process for a target deuteron initially at rest that is knocked-on by a projectile oxygen (triple scattering) is even larger. Since its initial energy is  $\sim$ zero, let us compare its energy gain relative to the initial energy of a projectile deuteron,  $E_{tf}/E_{pi} = (2M_0/(M_0 + m))^2 [1 + ((M_0 - m)/(M_0 + m))((M_{Ti} - m)/(M_{Ti} + m))]^2 = (2M_0/(M_0 + m))^2 (\tilde{U}/U) = 9.32$ . So we see that a much larger energy enhancement is possible than 2.23. There is an oversight in their final conclusion (page 1389) "We found that no deuterium atom ever moves more rapidly than about twice the velocity of the incident cluster..."<sup>7</sup> As shown here, significantly larger velocities are possible.

## IV. RESULTS

### IV.A. General

Using eqs. (4) and (6), the total fusion rates  $R_{calc}$  are calculated for both the  $(D_2O)_n^+$  and the  $(D)_n^+$  cases using  $kT_{max} = E_t/3n$  and  $kT_{max} = E_t/n$ , respectively. For each case, both complete thermalization ( $v_d = 0$  in eq. (11)) and partial thermalization ( $v_d \neq 0$  in eq. (11)) are considered. The calculated proton counts,

Table I: Comparison of our calculated  $D - D$  fusion counts,  $\tilde{R}_{calc} = \delta t \epsilon R_{calc}$ , with the experimental data of Beuhler *et al.* (BFF)<sup>1</sup> as deduced by Fallavier *et al.*<sup>11</sup> for the case of  $(D_2O)_n^+$ . The total kinetic energy  $E_t$  for the various  $(D_2O)_n^+$  clusters is 300 keV.  $R_{calc}$  is calculated using eqs. (14) and (16), and the detector efficiency  $\epsilon$  is assumed to be 10%. The target used in Ref. 1 is  $TiD$ . The target deuterium and titanium densities are assumed to be  $n_D = n_{Ti} = 5.7 \times 10^{22} \text{ cm}^{-3}$ .

n	Energy /D (eV)	Incident $D's(10^{15})$	$\tilde{R}_{calc}$ ( $v_d = 0$ )	$\tilde{R}_{calc}$ ( $v_d = v_0/\sqrt{2}$ )	$\tilde{R}_{calc}$ ( $v_d = v_0$ )	$\tilde{R}_{exp}$ (Ref. 1)
40	750	6.15	153	137	102	30
60	500	3.3	5.9	5.1	3.4	16
75	400	2.2	0.8	0.7	0.2	8

$\tilde{R}_{calc} = \delta t \epsilon R_{calc}$  (where  $\delta t$  is the acquisition time and  $\epsilon$  is the detector efficiency) are compared with the experimental results in Tables I and II for the  $(D_2O)_n^+$  and  $(D)_n^+$  cases, respectively.

As can be seen from Table I, our calculated proton counts agree with the results of BFF<sup>1</sup> to about an order of magnitude. Because there is expected to be less thermalization as the cluster size  $n$  decreases, our calculated counts probably overestimate the actual counts for smaller values of  $n$ . Furthermore, since Beuhler *et al.*<sup>2</sup> used  $(D_2O)_n^+$  beams that contained not just a single cluster size  $n$ , but a distribution in cluster size about  $n$ , we show below that our calculated counts become increasingly enhanced as the "mean" cluster size  $n$  increases due to the contribution of small clusters in the tail of the size distribution when we include the size distribution.

The proton counts for the experiment of Fallavier *et al.*<sup>11</sup> were also calculated with this model, and are seen to be very small (Table II). The  $(D)_n^+$  beams are not expected to have such a high degree of thermalization as the  $(D_2O)_n^+$  beams (see above), so calculations for the cases where half of the kinetic energy becomes thermal ( $v_d = v_0/\sqrt{2}$ ) and none of the kinetic energy becomes thermal ( $\delta$ -function velocity distribution) are included. Clearly these calculations show that the proton count (reaction rate) falls precipitously as the degree of thermalization decreases.

Because the parametrization of the cross section<sup>14</sup> used here is appropriate only for the scattering of bare deuterons, the effects of electron screening<sup>3,18,19</sup> in the target have not been taken into consideration in our calculations. However, at the energies relevant in these experiments,<sup>1,2,11</sup> electron screening is expected to increase the reaction rates only one order of magnitude or less.<sup>3,10,20</sup>

When we apply eqs. (4) and (6) to the case of large clusters (e.g.,  $(D_2O)_{1000}^+$ ), we find that even when complete thermalization ( $v_d = 0$ ) is assumed, the proton count will be  $\tilde{R}_{calc} = \delta t \epsilon R_{calc} = 4.04 \times 10^{-14}$  per  $10^{16}$   $D$ 's using the same value



Table II: for the case of  $(D)_n^+$ . The total kinetic energy per cluster is  $n$  times the energy per deuteron. The detector efficiency  $\epsilon$  is assumed to be 10%. The target deuteron and titanium densities are assumed to be  $n_D = n_{Ti} = 5.7 \times 10^{22} \text{ cm}^{-3}$ .  $R_{calc}(v_d = 0)$  and  $R_{calc}(v_d = v_0/\sqrt{2})$  are calculated using eqs. (14) and (16).  $\tilde{R}_{calc}(mono)$  is calculated with a delta function (monoenergetic) distribution using eq. (19). The experimental values of  $\tilde{R}_{exp}$  are all zero.<sup>11</sup>

n	Energy /D (eV)	Incident D's( $10^{15}$ )	Target	$\tilde{R}_{calc}(v_d = 0)$	$\tilde{R}_{calc}(v_d = v_0/\sqrt{2})$	$\tilde{R}_{calc}(mono)$ ( $\delta$ -function)
200	750	6.15	$(CD_2)_n$	$1.71 \times 10^{-2}$	$1.64 \times 10^{-3}$	$1.00 \times 10^{-13}$
200	500	3.68	$(CD_2)_n$	$1.78 \times 10^{-4}$	$9.40 \times 10^{-6}$	$4.32 \times 10^{-19}$
200	500	2.96	$TiD_{1.7}$	$1.43 \times 10^{-4}$	$7.56 \times 10^{-6}$	$5.02 \times 10^{-19}$
250	400	2.27	$TiD_{1.7}$	$9.05 \times 10^{-6}$	$3.30 \times 10^{-7}$	$2.50 \times 10^{-22}$
300	400	2.15	$TiD_{1.7}$	$8.57 \times 10^{-6}$	$3.13 \times 10^{-7}$	$1.68 \times 10^{-22}$

of  $\epsilon = 0.1$  as above. The equivalent proton count per  $10^{16}$  D's from BFF<sup>1</sup> is  $\tilde{R}_{exp} \approx 2.4$ , which is some 14 orders of magnitude larger than  $\tilde{R}_{calc}$  (see Fig. 2). Therefore, impact thermalization by itself is inadequate to explain the large cluster (low-energy deuteron) results of Beuhler *et al.*,<sup>1,2</sup> and thus some other mechanism—e.g., contaminants, low energy  $D - D$  fusion resonances,<sup>4</sup> cluster size distribution (“mass dispersion”),<sup>2,7</sup> and/or electron screening<sup>3,9,19</sup>—must also contribute to explain these results. However, when compared to the traditionally calculated mono-energetic rate (i.e., when  $f(\vec{v}) = \delta(\vec{v} - \vec{v}_0)$ ),  $\tilde{R}_{mono} = 1.08 \times 10^{-102}$  per  $10^{16}$  D's, the use of a velocity distribution such as eq. (11) in  $\tilde{R}_{calc}$  does account for most of the discrepancy between  $\tilde{R}_{mono}$  and  $\tilde{R}_{exp}$ , as can be seen from Fig. 2.

In Fig. 3, we include the effects of the mass distribution. The dashed line is the case with no mass distribution (eq. (14)), the solid line is the case with  $\hat{n} = \bar{n}/2$  for the cutoff of the distribution integral (eq. (24)), and the dotted line is the case with  $\hat{n} = 1$  for the cutoff of the distribution integral (i.e., no cutoff). Note that for  $\bar{n} = 1000$  (corresponding closely to the cold fusion case, with  $E_{cm} \approx 10$  eV), the inclusion of a mass distribution with  $\hat{n} = \bar{n}/2$  (the situation claimed by Beuhler *et al.*<sup>2</sup>) increases  $\tilde{R}_{calc}$  from  $5.04 \times 10^{-14}$  to  $2.22 \times 10^{-11}$  per  $1.25 \times 10^{16}$ —a factor of  $\sim 400$ , but still some eleven orders of magnitude below  $\tilde{R}_{exp}$ . However, for the case of a mass distribution with no cutoff,  $\hat{n} = 1$ ,  $\tilde{R}_{calc}$  is now 8.59, about  $\sim 4$  times larger than  $\tilde{R}_{exp}$ . This indicates that if there is some cluster breakup between the mass spectrometer and the entrance to the accelerator column in the experiments of Beuhler *et al.* that “replaces” a portion of the “missing” part of  $f(n, v)$  below  $\hat{n} = \bar{n}/2$ , then  $\tilde{R}_{calc}$  will be enhanced at least by several orders of magnitude. However, since at  $n \approx 100$ ,  $\tilde{R}_{calc}(\hat{n} = 1)$  is still more than an order of

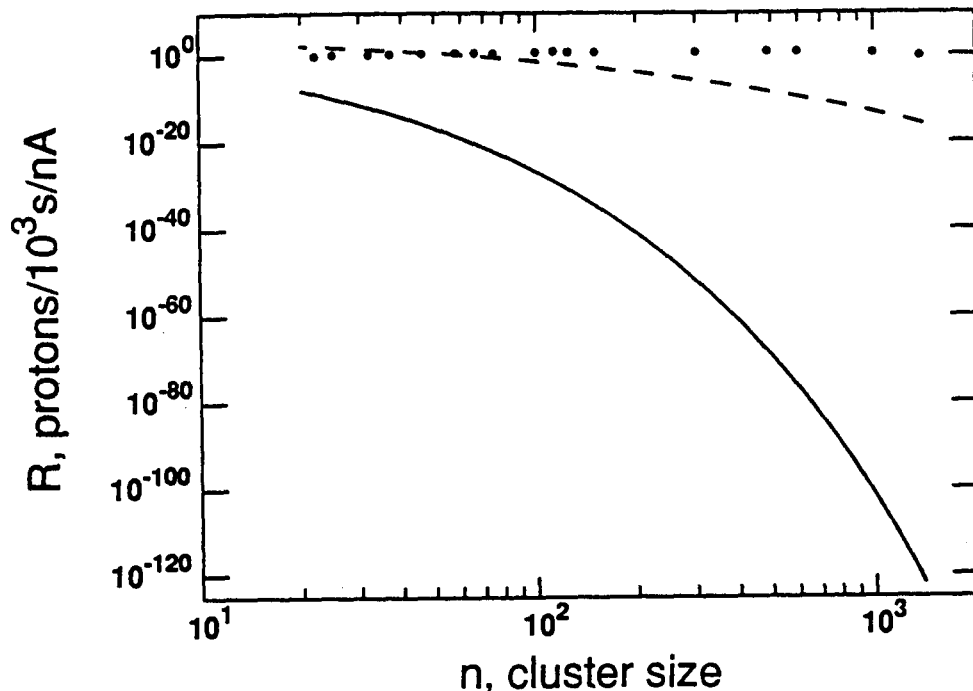


Figure 2: Proton counting rate,  $\tilde{R} = \delta t \epsilon R$ , as a function of cluster size,  $n$ . The units of  $\tilde{R}$ ,  $10^{-3} s^{-1} (nA)^{-1}$ , is equivalent to  $1.25 \times 10^{12} n$  clusters when  $\epsilon = .1$  and  $\delta t = 10^3 s$ . The data (dots) are from Fig. 3 of Ref. 1. For comparison, we show the results for a conventional nuclear theory calculation assuming monoenergetic deuterons (solid line, eq. (19)), and for a Maxwell-Boltzmann (M-B) distribution of completely thermalized deuterons (dashed line, eq. (14) with eq. (18) for  $v_d = 0$ ).

magnitude below  $\tilde{R}_{exp}$ , there are still contributing processes not yet considered to be included in our theory, which may also be important to cold fusion.

#### IV.B. Low-Energy Resonances

One possible candidate for the missing contribution is the existence of low-energy  $D - D$  fusion cross-section resonances.<sup>4</sup> An example of the effect of such a resonance is shown in Fig. 4, where we have added a Breit-Wigner resonance, eq. (10), to eq. (8), and included a mass-velocity distribution, eq. (24), with  $\hat{n} = \bar{n}/2$  (note that for simplicity, we do not consider interference terms here). The parameters of this Breit-Wigner resonance are as follows: the peak energy of the resonance,  $E_r^{(pT)}$ ; the total width of the resonance,  $\Gamma$ ; and the partial width of the  $D - D$  entrance channel,  $\Gamma_{ent}$ , are all taken to be 400 eV; the partial width of the  $p - T$  exit channel,  $\Gamma^{(pT)}$ , is set to  $2.03 \times 10^{-8}$  eV. To insure that the cross-section converges to zero as  $E \rightarrow 0$ , we have multiplied eq. (10) by a factor

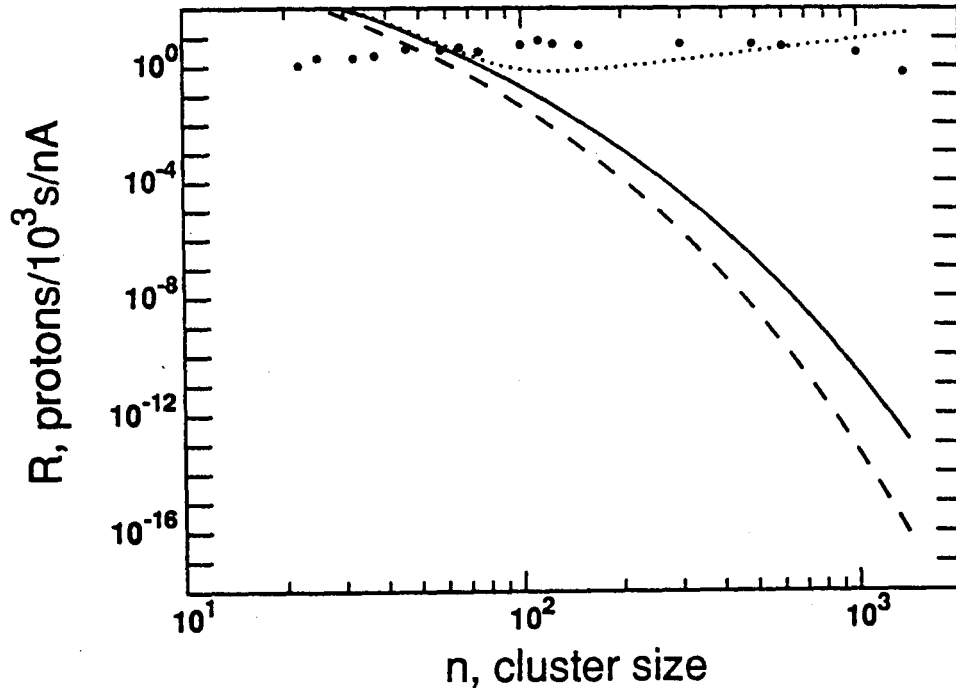


Figure 3: Proton counting rate,  $\bar{R} = \delta t \epsilon R$ , as a function of cluster size,  $n$ . Here we consider the effect of the cluster size distribution, eq. (20). The dashed line is the same result as the dashed line in Fig. 2: a M-B distribution with no cluster size distribution. The solid line is for a cluster size distribution cutoff of  $\hat{n} = \bar{n}/2$  ( $\bar{n}$  being the peak of the cluster size distribution), and the dotted line is for  $\hat{n} = 1$ . Also see Fig. 2 for more information.

of  $\exp(1 - E_r^{(pT)}/E)$  when  $E < E_r^{(pT)}$ . Observe that the resonance (with the low-energy suppression factor  $\exp(1 - E_r^{(pT)}/E)$ ) fits  $\bar{R}_{exp}$  to within a factor of 4 for  $n \gtrsim 50$ , peaks at  $n \approx 300$  and drops off slowly as  $n$  becomes very large. One should, however, consider this as only speculative, since there is no direct experimental evidence for the existence of such a resonance. Furthermore, the values of the resonance parameters ( $\Gamma$ ,  $\Gamma^{(\alpha)}$ , etc.) are extremely sensitive to the other contributions to  $\bar{R}$  (e.g., mass and velocity distribution). Nonetheless, the possibility that such low-energy  $D-D$  resonances might exist also relates directly to cold fusion and adds further emphasis to the need for conventional very low-energy ( $< 1$  keV)  $D-D$  scattering experiments.<sup>4</sup>

#### IV.C. Other Effects

Other possible candidates for the missing contribution are 3D Efimov resonances,<sup>21</sup> the aforementioned much higher multiple-scattering probabilities for the low-energy deuterons from large clusters than for the high-energy deuterons from

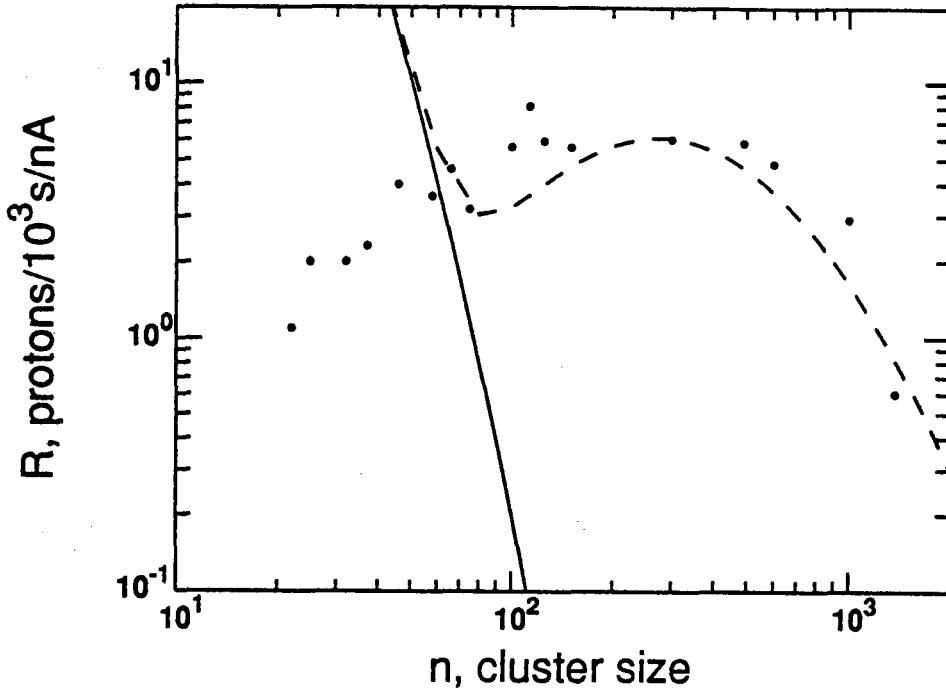


Figure 4: Proton counting rate,  $\tilde{R} = \delta t \epsilon R$ , as a function of cluster size,  $n$ . Here we consider the effect of a very low-energy Breit-Wigner (B-W) resonance in the D-D fusion cross-section. The solid line is the same result as the solid line in Fig. 3 (for eq. (20) with a cluster size cutoff of  $\hat{n} = \bar{n}/2$ ), and the dashed line is also the same result, but with the B-W resonance included. See the text for the parameters of the B-W resonance and Fig. 2 for more information.

small clusters, target/cluster compression, and the effects of final state interactions.

Finally, observe from Figs. 3 and 4 that for  $n \lesssim 50$ ,  $\tilde{R}_{calc} \gg \tilde{R}_{exp}$ . This almost certainly reflects the fact that for small clusters, there are few deuterons, each of which has energy  $E_0$  (or  $E_D$ ) that takes up a substantial portion of  $E_t$ . Therefore, the use of M-B distributions is probably inappropriate for modeling the velocity distribution of very small clusters, and one must use a distribution that is discretized in frequency, and severely cut off in energy to reflect conservation of  $n$  and  $E_t$ .

## V. PREDICTIONS AND PROPOSED TESTS

In the following, several experimental measurements are proposed to test the predictions of our theoretical model.

## V.A. Degree of Thermalization

The degree of thermalization in  $(D_2O)_n^+$  and  $(D)_n^+$  can be tested experimentally using the same value of  $kT_{max}$  for both cases (through the equipartition theorem), i.e.

$$3kT_{max}/2 = E_t((D)_{n_1}^+)/n_1 = E_t((D_2O)_{n_2}^+)/3n_2. \quad (25)$$

The experimental condition imposed by eq. (25) can be achieved in several ways: (a) set  $E_t((D)_{n_1}^+) = E_t((D_2O)_{n_2}^+) = E_t$  and hence  $n_1 = 3n_2$ , (b) set  $n_1 = n_2$  and hence  $E_t((D_2O)_{n_2}^+) = 3E_t((D)_{n_1}^+)$ , etc.. The observed  $D - D$  fusion rates are expected to satisfy  $\tilde{R}_{exp}((D_2O)_{n_2}^+) > \tilde{R}_{exp}((D)_{n_1}^+)$ , for both cases (a) and (b), if impact thermalization of the cluster due to multiple collisions is a correct interpretation for cluster-beam fusion experiments.<sup>1,2,11</sup> For example, the null result  $\tilde{R}_{exp}((D)_{200}^+) = 0$  fusion reactions for a total number of  $2.96 \times 10^{15}$  and  $3.68 \times 10^{15}$  incident deuterons as observed by Fallavier *et al.*<sup>11</sup> (see Table II) with an energy per deuteron of  $mv_0^2/2 = 500$  eV (or  $E_t = 100$  keV) and  $3kT_{max}/2 = 500$  eV can be compared with the positive result  $\tilde{R}_{exp}((D_2O)_{150}^+) \approx 1.4$   $D(D,p)T$  fusion reactions for a total number of  $3.3 \times 10^{15}$  incident deuterons as deduced from the fusion rate of 0.8 proton counts per  $10^3$  seconds per  $nA$  observed by BFF<sup>1</sup> (their figure 4) with  $E_t = 225$  keV or  $mv_0^2/2 = 150$  eV and  $3kT_{max}/2 = 500$  eV. Without impact thermalization ( $f(\vec{v}) = \delta(\vec{v} - \vec{v}_0)$ ), it is expected that  $\tilde{R}_{calc}^{\delta}((D)_{200}^+) \gg \tilde{R}_{calc}^{\delta}((D_2O)_{150}^+)$  since  $\sigma(500 \text{ eV}) \gg \sigma(150 \text{ eV})$ . The fact that  $\tilde{R}_{exp}((D_2O)_{150}^+) > \tilde{R}_{exp}((D)_{200}^+)$  indicates that the degree of impact thermalization is much greater for the  $((D_2O)_{150}^+)$  case<sup>1</sup> than for the  $((D)_{200}^+)$  case.<sup>11</sup>

## V.B. Low-Energy $D - D$ Cross-Sections and Branching Ratio

If low-energy  $D - D$  resonances exist, the branching ratio of reaction (6) to reaction (7) is expected to deviate from the expected value of unity. The branching ratio,  $R_b \approx \tilde{R}^T/\tilde{R}^{3He}$ , extracted from the measured  $^3H$  and  $^3He$  spectra by Beuhler *et al.*<sup>2</sup> is  $1.2 \pm 0.2$  for the case of a 275 keV  $(D_2O)_{115}$  beam incident on a  $(C_2D_4)_n$  target, corresponding to an incident deuteron LAB kinetic energy of 0.239 keV/ $D$ . Thus their extracted value represents the lowest energy measurement ever made of  $R_b$ . Since even lower energy measurements may yield further deviation from  $R_b \approx 1$ , it would be worthwhile to measure  $R_b$  as a function of  $n$  for larger cluster sizes,  $115 < n \lesssim 1000$ .

## V.C. Target Deuteron Thermalization

Since we interpret the thermalization of target deuterons initiated by heavy atoms in the projectile cluster (such as the knock-on process followed by a double backscattering of the knocked-on target deuteron from a heavy target atom and

a projectile oxygen) as a separate process from the (non-equilibrium) thermalization of projectile deuterons described by  $f(\vec{v})$  of eq. (11), we predict that the case of a  $(X_\beta)_{n_1}$  incident on a  $(C_2D_4)_n$  target ( $X$  is an atom comparable to or heavier than  $O$ ) will produce a similar positive result as for the case of 300 keV  $(H_2O)_{115}$  on  $(C_2D_4)_n$  carried out by Beuhler *et al.*<sup>2</sup> We propose the proton measurement for  $(X_\beta)_{n_1}$  clusters incident (such as  $C_{60}$ )<sup>22</sup> on  $(C_2D_4)_n$  to test the above idea with appropriate choices of  $X$  and its incident energy comparable ( $\sim 2.32$  keV LAB per  $X$ ) to that of  $O$  in the case of 300 keV  $(H_2O)_{115}$  on  $(C_2D_4)_n$ . In addition, we propose a blank experiment in which the  $(C_2D_4)_n$  target is replaced by  $(C_2H_4)_n$  in order to rule out the possibility of systematic errors in previous experiments using  $(H_2O)_{115}$  projectiles and  $(C_2D_4)_n$  targets.

## VI. CONCLUSION

In conclusion, the cluster-transport impact fusion model discussed in this paper can simultaneously describe the positive  $(D_2O)_n^+$  small  $n$  cluster results of Beuhler *et al.*,<sup>1,2</sup> and the negative  $(D)_n^+$  cluster results of Fallavier *et al.*<sup>11</sup> for cluster impact fusion. The anomalously high fusion rate by a factor of  $\sim 10^{100}$  observed by Beuhler *et al.*<sup>1,2</sup> for a small CM energy of 15 eV/ $D$  resembles the high fusion rates reported for cold fusion.<sup>5,6</sup>

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