THE ROLE OF VELOCITY DISTRIBUTION IN COLD DEUTERIUM-DEUTERIUM FUSION

COLD FUSION

TECHNICAL NOTE

KEYWORDS: *cold D-D fusion, velocity distribution*

ROBERT A. RICE, GARY S. CHULICK, YEONG E. KIM,
and JIN-HEE YOON *Purdue University, Department of Physics*
West Lafayette, Indiana 47907

Received February 9, 1990
Accepted for Publication April 2, 1990

Reaction rates from recent electrochemical fusion experiments have been found to be as many as seventy orders of magnitude larger than those obtained from simple calculations involving an extrapolated low-energy deuterium-deuterium (D-D) cross section and a sharp velocity distribution. However, if an appropriate Maxwell-Boltzmann velocity distribution is used in place of the conventional sharp (monoenergetic) velocity distribution, the calculated reaction rate increases by as much as fifty to sixty orders of magnitude. Furthermore, the center-of-mass energy at which the D-D cross section is evaluated for given D-D energy is much larger than that used in the conventional calculations due to the higher energy components in the Maxwell-Boltzmann distribution. Finally, the above results are not significantly affected if a reasonable high-energy cutoff E_c is included in the velocity distribution.

I. INTRODUCTION

Recently, a surface reaction mechanism[1,2] was proposed as a consistent and plausible explanation for tritium and neutron production and excess heat generation above that due to the electrode reaction reported by Fleischmann and Pons[3] and others[4–8] in their electrolysis experiments. In the surface reaction mechanism, deuterium-deuterium (D-D) fusion takes place in the surface zone of a palladium cathode where whiskers of metal deuterides (PdD and/or LiD) are formed by electrolysis. These whiskers occupy a surface zone of $\gtrsim 10$-μm thickness, where most D_2 gas bubbles are formed from the dissociation of D_2O. Depending on the electrolysis conditions, many spherical and hemispherical D_2 gas bubbles of various sizes (radii ranging from a few micrometres to a few millimetres) are produced in the surface whisker zone where they stay for a certain time duration before they move from the cathode. Many of these D_2 gas bubbles in the surface whisker zone have whiskers protruding into them, creating field emission potentials around the tips of the whiskers. The average potential in each D_2 bubble is expected to be approximately that of the applied potential of the electrolysis cell, but the electric field near the whisker tips can be several orders of magnitude larger than the average field, as is well known from field emission studies. Thus, D^+ ions in the bubble gain kinetic energies with a statistical distribution that depends on the bubble size and values of the widely varying electric field inside the bubble. When the applied potential is ~ 10 V, the average laboratory kinetic energy of the D^+ ions in each bubble is expected to be ~ 10 eV.

It has been argued[1,2] that the D-D fusion rate with a Maxwell-Boltzmann D^+ velocity distribution can become very large at low energies compared to that with a sharp velocity distribution. In this technical note, we present results of our detailed calculations of the D-D fusion rates using both a sharp (δ function) velocity distribution and a statistical distribution with and without a cutoff in the high-velocity components. In Sec. II, definitions and expressions for the D-D fusion cross section and rate are described. In Sec. III, the calculated results are presented and discussed. Finally, Sec. IV contains a summary.

II. THEORETICAL FORMULATION

The two dominant channels for D-D fusion are

$$D + D \rightarrow {}^3H(1.01\ MeV) + p(3.02\ MeV) \quad (1a)$$

and

$$D + D \rightarrow {}^3He(0.82\ MeV) + n(2.45\ MeV)\ . \quad (1b)$$

Reaction (1a) is not "real" fusion but a neutron-transfer reaction, while reaction (1b) is a fusion reaction in which two protons are fused to form a ^{3}He nucleus. Because of the complexity of the four-nucleon system, no rigorous theoretical calculations of the D-D fusion rates and branching ratios have been carried out at low positive energies. Since there are no direct measurements of the D-D fusion cross sections $\sigma(E)$ for reactions (1a) and (1b) below $E_D \lesssim 4$ keV (laboratory frame), $\sigma(E)$ has to be extrapolated using the measured values of $\sigma(E)$ for $E_D \gtrsim 4$ keV, as is conventionally done in astrophysical calculations[9,10] for temperatures of $kT \gtrsim 1.5$ keV. In our case, an extrapolation down to the order of 10 eV is required. We adopt the extrapolation method used in astrophysical calculations[9,10] in the following. Furthermore, we assume equal branching ratios for reactions (1a) and (1b) for simplicity of calculation due to the lack of experimental data at extremely low energies.

II.A. Maxwell-Boltzmann Distribution

Since the precise form of the D^+ velocity distribution in electrolysis experiments is not known at present, we assume a Maxwell-Boltzmann distribution with and without a cutoff for high-velocity components. We use a velocity cutoff in the Maxwell-Boltzmann distribution to simulate the situation in the D_2 gas bubbles, where the high-energy tail is probably truncated. The temperature term kT is replaced by the "average" kinetic energy $E_{D\text{-}D}$, in the center-of-mass (CM) D-D frame, which is related to the most probable velocity v (CM) by $E_{D\text{-}D} = (M/2)v^2$ (CM) with the reduced mass $M = M_D/2$.

For a Maxwell-Boltzmann velocity distribution, the D-D fusion rate, Λ (s^{-1}/D-D pair), for reaction (1a) or (1b) is given by[9,10]

$$\Lambda(E_{D\text{-}D}) = \frac{n_D}{2}\langle\sigma v\rangle \ , \tag{2}$$

with

$$\langle\sigma v\rangle = \frac{(8/\pi)^{1/2}}{M^{1/2}(E_{D\text{-}D})^{3/2}}\int_0^{E_c}\sigma(E)E\exp(-E/E_{D\text{-}D})\,dE \ , \tag{3}$$

where the cross section $\sigma(E)$ is parameterized as (E is in the CM frame)

$$\sigma(E) = \frac{S(E)}{E}\exp[-(E_G/E)^{1/2}] \ , \tag{4}$$

which is the conventional form assuming nonresonant charged-particle reactions for reactions (1a) and (1b). Here, E_G is the "Gamow energy" given by $E_G = (2\pi\alpha Z_D Z_D)^2 Mc^2/2$ or $E_G^{1/2} \approx 31.28\ (\text{keV})^{1/2}$ for the reduced mass $M \approx M_D/2$. The extrapolated values of the S factors for reactions (1a) and (1b) are nearly equal at $E \approx 0$ [$S(E \approx 0) \approx 55$ keV·b], although $S(E)$ for reaction (1b) is slightly larger than $S(E)$ for reaction (1a) for $E \gtrsim 20$ keV (Ref. 11). The deuterium density n_D is assumed to be $\sim 6 \times 10^{22}\ \text{cm}^{-3}$. In Eq. (3), E_c is the upper integration limit beyond which the high-velocity components are cut off.

With a Taylor series expansion of $S(E)$ as given in Ref. 9,

$$S(E) \approx S(0) + S'(0)E + \frac{1}{2}S''(0)E^2 \ , \tag{5}$$

the integral in Eq. (3) can be shown to approximate for the case of no cutoff, $E_c \to \infty$,

$$\langle\sigma v\rangle = \left(\frac{2}{M}\right)^{1/2}\frac{\Delta E_0}{(E_{D\text{-}D})^{3/2}}S_{eff}\exp(-\tau) \ , \tag{6}$$

where

$$\Delta E_0 = 4(E_0 E_{D\text{-}D}/3)^{1/2} \ ,$$

$$\tau = 3E_0/E_{D\text{-}D} \ ,$$

and

$$S_{eff} = S(0)\left[1 + \frac{5}{12\tau} + \frac{S'(0)}{S(0)}\left(E_0 + \frac{35}{36}E_{D\text{-}D}\right) + \frac{1}{2}\frac{S''(0)}{S(0)}\left(E_0^2 + \frac{89}{36}E_0 E_{D\text{-}D}\right)\right] \ ,$$

with

$$E_0 = (E_G^{1/2}E_{D\text{-}D}/2)^{2/3} \ .$$

For a finite cutoff energy E_c in Eq. (3), the results for $\langle\sigma v\rangle$ are obtained by carrying out the integration numerically. The integrand of Eq. (3) is shown in Fig. 1. The average kinetic energy $E_{D\text{-}D}$ (CM) is expected to be ~ 10 eV when the applied potential is ~ 20 V, as in a typical electrolysis experiment.

II.B. Sharp Velocity Distribution

For a sharp velocity distribution, which has been used to argue against the possibility of D-D fusion at room temperature, the D-D fusion rate (s^{-1}/D-D pair) Λ_δ is given by

$$\Lambda_\delta(E) = \frac{n_D}{2}\sigma(E)v\ (\text{CM}) \ , \tag{7}$$

where

$$E = E_{D\text{-}D} = \frac{M}{2}v^2\ (\text{CM}) = \frac{M_D}{4}v^2(lab) = \frac{1}{2}E_D(lab) \ .$$

III. RESULTS

The extrapolated cross section $\sigma(E)$ for D-D fusion for both reactions (1a) and (1b) calculated from Eqs. (4) and (5) using parametric values given in Ref. 9 is plotted as a function of $E_{D\text{-}D}$ (CM) in Fig. 2 ($E_{D\text{-}D} \leq 20$ eV) and Fig. 3 ($E_{D\text{-}D} \leq 200$ eV). The extrapolated values of $\sigma(E)$ are used to calculate the D-D fusion rates Λ and Λ_δ using Eqs. (2) and (7), respectively. The calculated results for Λ (upper curve) and Λ_δ (lower curve) for the case of $E_c \to \infty$ are shown in Fig. 4 ($E_{D\text{-}D} \leq 20$ eV) and in Fig. 5 ($E_{D\text{-}D} \leq 200$ eV). As can be seen from Figs. 4 and 5, the calculated rate Λ with the Maxwell-Boltzmann distribution [Eq. (2)] becomes astronomically larger than the rate Λ_δ calculated with a sharp distribution [Eq. (7)], as $E_{D\text{-}D}$ decreases. At $E_{D\text{-}D} \approx 30$ eV, the ratio of Λ to Λ_δ is $(\Lambda/\Lambda_\delta) \approx 10^{52}$, and becomes even greater as $E_{D\text{-}D}$ decreases below 30 eV.

The effect of introducing a cutoff for higher velocity components in the Maxwell-Boltzmann distribution is investigated by varying the upper limit E_c of the integration in Eq. (3). The calculated results for $\Lambda(E_{D\text{-}D})$ are plotted as a function of $E_c \leq 1$ keV for the cases of "average" kinetic

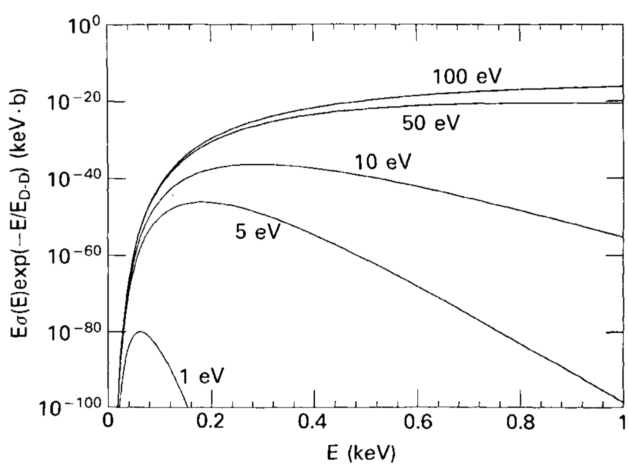

Fig. 1. The integrand of Eq. (3) for five different values of the average center-of-mass kinetic energy $E_{D\text{-}D}$.

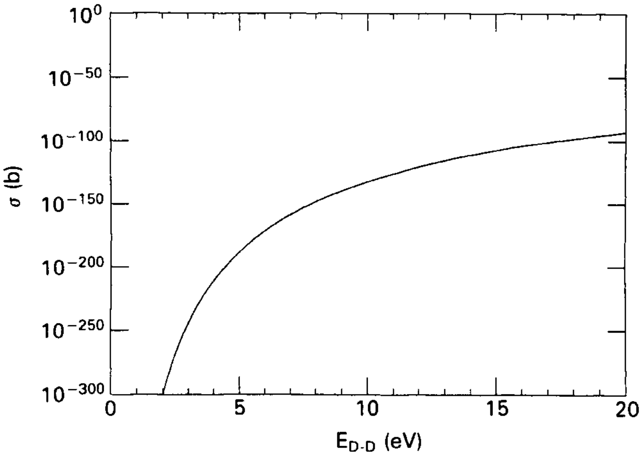

Fig. 2. The extrapolated cross section for reactions (1a) and (1b) for $E_{D\text{-}D} \leq 20$ eV.

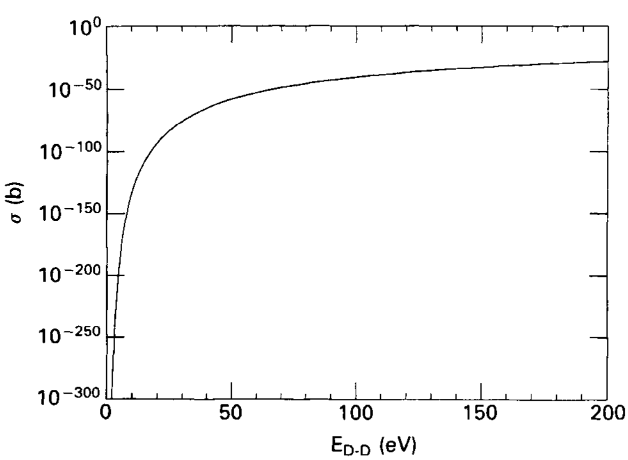

Fig. 3. The extrapolated cross section for reactions (1a) and (1b) for $E_{D\text{-}D} \leq 200$ eV.

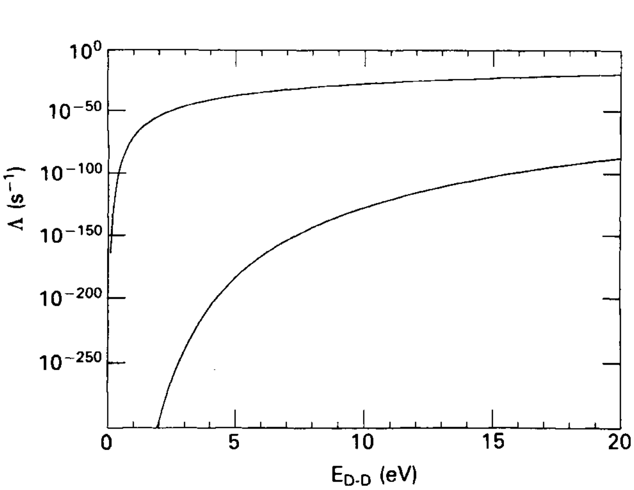

Fig. 4. The calculated reaction rate Λ (s^{-1}/D-D pair) for reactions (1a) and (1b) as a function of $E_{D\text{-}D} \leq 20$ eV. The upper curve is the result of using the Maxwell velocity distribution, while the lower curve is the result of using a δ function velocity distribution.

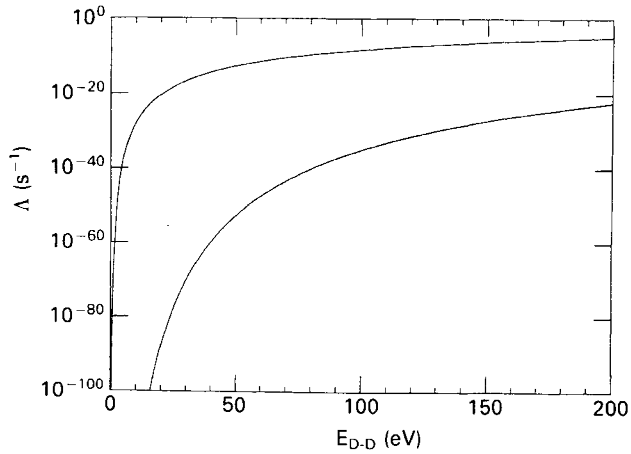

Fig. 5. Same as Fig. 4, but for $E_{D\text{-}D} \leq 200$ eV.

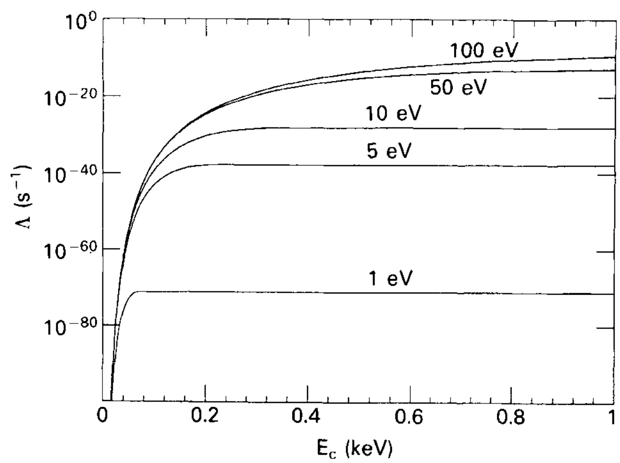

Fig. 6. The calculated reaction rate for reactions (1a) and (1b) including a cutoff at energy E_c in the Maxwell velocity distribution. The result is shown as a function of E_c for five different values of the average CM energy $E_{D\text{-}D}$.

energies of $E_{D\text{-}D} = 1$, 5, 10, 50, and 100 eV in Fig. 6. As can be seen in Fig. 6, the results for Λ calculated with cutoff distributions are not significantly affected if a reasonably high-energy cutoff is used in the velocity distribution.

The D-D fusion rate for reaction (1a), $\Lambda^T_{exp}(1a) \approx 10^{-19}$ s^{-1}, inferred by Fleischmann and Pons[3] from the measurement of tritium production, and also the D-D fusion rate for reaction (1b), $\Lambda^n_{exp}(1b) \approx 10^{-23}$ s^{-1}, obtained by Jones et al.[4] are often criticized as being impossible or incorrect when compared to estimates of Λ in bulk matter (an upper limit of $\Lambda \lesssim 10^{-47}$ s^{-1}) (Ref. 12) or to the result of Λ_δ shown in Figs. 4 and 5 (lower curves). Our results for Λ (upper curves) shown in Figs. 4 and 5 indicate that $\Lambda^n_{exp}(1b) \approx (10^{-23}/s)^4$ and $\Lambda^T_{exp} \approx (10^{-19}/s)^3$ are consistent with calculated values of $\Lambda(E_{D\text{-}D}) \approx 10^{-23}$ s^{-1} and $\Lambda(E_{D\text{-}D}) \approx 10^{-19}$ s^{-1} for $E_{D\text{-}D} \approx 15$ eV and 20 eV, respectively. Therefore, the claimed values of $\Lambda^n_{exp}(1b) \approx (10^{-23}/s)^4$ and $\Lambda^T_{exp}(1a) \approx (10^{-19}/s)^3$ are physically acceptable values for the D-D fusion rate in electrolysis experiments if the applied potentials are 30 and 40 V, respectively.

To match the D-D fusion rate $\Lambda(E_{D\text{-}D})$ from reaction (1a) to the rate $\Lambda_{exp}^{heat}(1a) \approx (10^{-10}/s)^3$, inferred by excess heat measurements,[3,5,6] an average kinetic energy of $E_{D\text{-}D} \approx 75$ eV is needed. At present, there are no known physical mechanisms which enable D^+ ions to attain an average D^+ kinetic energy $E_{D\text{-}D} > 20$ eV when the applied potential is <40 V in electrolysis experiments. However, if, in the future, experimentally measured values of $\sigma(E)$ for reactions (1a) and/or (1b) at low energies turn out to be larger than the extrapolated values by a factor of $\sim 10^{10}$ (which is a small scaling factor for Λ when compared to $\Lambda/\Lambda_\delta \approx 10^{52}$ at $E_{D\text{-}D} \approx 30$ eV) (Ref. 13), then an increased rate of $\Lambda(E_{D\text{-}D}) \approx 10^{-10}\,s^{-1}$ would result from a Maxwell-Boltzmann velocity distribution with an average kinetic energy of $E_{D\text{-}D} \approx 20$ eV, a value expected when a potential of 40 V is applied in electrolysis experiments.

IV. SUMMARY

It is shown that the low-energy D-D fusion rates $\Lambda(E_{D\text{-}D})$ calculated with a Maxwell-Boltzmann D^+ velocity distribution are astronomically larger (a factor of 10^{52} at $E_{D\text{-}D} \approx 30$ eV) than the conventional estimates, $\Lambda_\delta(E_{D\text{-}D})$, calculated with a sharp distribution, as shown in Figs. 4 and 5.

The claimed values of $\Lambda_{exp}^{T}(1a) \approx (10^{-19}/s)^3$ and $\Lambda_{exp}^{n}(1b) \approx (10^{-23}/s)^4$ are consistent with the surface reaction mechanism when the D^+ flux with a Maxwell-Boltzmann velocity distribution is maintained at average kinetic energies of $E_{D\text{-}D} = 20$ and 15 eV, respectively; these energies correspond to an applied potential of 40 and 30 V, respectively, for electrolysis experiments.

For the claimed value of $\Lambda_{exp}^{heat} \approx (10^{-10}/s)$ (Refs. 3, 5, and 6), an additional increase of $\sim 10^{10}$ in the value of $\sigma(E)$ is needed to obtain a value of $\Lambda(E_{D\text{-}D}) \approx 10^{-10}/s$ at $E_{D\text{-}D} \approx 20$ eV.

REFERENCES

1. Y. E. KIM, "Nuclear Theory Hypotheses for Cold Fusion," *Proc. NSF/EPRI Workshop Anomalous Effects in Deuterated Metals*, Washington, D.C., October 16–18, 1989.

2. Y. E. KIM, "Fission-Induced Inertial Confinement Hot Fusion and Cold Fusion with Electrolysis," *Laser Interaction and Related Plasma Phenomena*, Vol. 9, H. HORA and G. H. MILEY, Eds., Plenum Press, New York (to be published).

3. M. FLEISCHMANN and S. PONS, "Electrochemically Induced Nuclear Fusion," *J. Electroanal. Chem.*, **261**, 301 (1989); and Errata, **263**, 187 (1989).

4. S. E. JONES et al., "Observation of Cold Nuclear Fusion in Condensed Matter," *Nature*, **338**, 737 (Apr. 27, 1989).

5. A. J. APPELBY, S. SRINIVASAN, Y. J. KIM, O. J. MURPHY, and C. R. MARTIN, "Evidence for Excess Heat Generation Rates During Electrolysis of D_2O in LiOD Using a Palladium Cathode—A Microcalorimetric Study," *Proc. Workshop Cold Fusion Phenomena*, Santa Fe, New Mexico, May 23–25, 1989.

6. A. BELZNER, U. BISCHLER, G. CROUCH-BAKER, T. M. GUR, G. LUCIER, M. SCHREIBER, and R. HUGGINS, "Two Fast Mixed-Conductor Systems: Deuterium and Hydrogen in Palladium-Thermal Measurements and Experimental Considerations," *Proc. Workshop Cold Fusion Phenomena*, Santa Fe, New Mexico, May 23–25, 1989.

7. K. L. WOLF, N. J. C. PACKHAM, D. R. LAWSON, J. SHOEMAKER, F. CHENG, and J. C. WASS, "Neutron Emission and the Tritium Content Associated with Deuterium Loaded Palladium and Titanium Metals," *Proc. Workshop Cold Fusion Phenomena*, Santa Fe, New Mexico, May 23–25, 1989.

8. P. K. IYENGAR, "Cold Fusion Results in BARC Experiment," *Proc. 5th Int. Conf. Emerging Nuclear Energy Systems (ICENES V)*, Karlsruhe, FRG, July 3–6, 1989.

9. W. A. FOWLER, G. R. CAUGHLAN, and B. A. ZIMMERMAN, "Thermonuclear Reaction Rates," *Ann. Rev. Astr. Astrophys.*, **5**, 525 (1967); "Thermonuclear Reaction Rates II," *Ann. Rev. Astr. Astrophys.*, **13**, 69 (1975).

10. M. J. HARRIS, W. A. FOWLER, G. R. CAUGHLAN, and B. A. ZIMMERMAN, "Thermonuclear Reaction Rates III," *Ann. Rev. Astr. Astrophys.*, **21**, 165 (1983).

11. A. KRAUSS, H. W. BECKER, H. P. TRAUTVETTER, and C. ROLFS, "Low-Energy Fusion Cross Sections of D + D and D + ^{3}He Reactions," *Nucl. Phys.*, **A465**, 150 (1987).

12. A. J. LEGGETT and G. BAYM, "Exact Upper Bound on Barrier Penetration Probabilities in Many-Body Systems: Application to Cold Fusion," *Phys. Rev. Lett.*, **63**, 191 (1989).

13. Y. E. KIM, "Cross Section for Cold Deuterium-Deuterium Fusion," *Fusion Technol.*, **17**, 507 (1990).