Bass, R.W., *Parmenter's Fundamental Breakthrough Contributions.* Infinite Energy, 1998. **4**(21): p. 45.

Parmenter's Fundamental Breakthrough Contributions

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Noting Dr. Parmenter's acknowledgment to me at the end of his seminal paper, Dr. Mallove has asked me for a prefatory critique. Frankly I feel like a kindergarten finger-painting dauber asked to appraise a Rembrandt! In fact, in 1994 I applied seriously for a humble programmer's job at the Univ. of Arizona in hopes that by moving to Tucson I might be able to audit some of Parmenter's courses: I am awed by his mastery of the three-dimensional details, not only of Quantum Mechanics (QM) [which I know only as a 1-D point-particle theory] but of Quantum Electrodynamics (QED), Nuclear Physics, and Solid-State Physics. I accepted this assignment only in hopes of nudging people like Dr. Barry Merriman of UCLA and Dr. Jim Peebles of Princeton to consider Parmenter's contributions with the serious care which they manifestly deserve. I'd also hope that in the next issue of *IE* we receive comments on this milestone theoretical *tour de force* by all of the dozen other expert theoreticians mentioned below.

Recently I sent a copy of the Parmenter paper, together with an implied apology for the short deadline, to the pre-eminent astrophysicist/cosmologist and current *Albert Einstein Professor of Science* at Princeton University, Dr. P.J.E. Peebles, whose opinion seems to me to be the key-stone of conservative skepticism about CF, as explained in (b) below. I have just received this e-mail from him:

>Hello, Robert Bass,

I did look over the paper by Parmenter. He got to the right
physics in equation (25), but I believe messed up in the
computation of |Psi (R)| equation 27), and I can make no sense of his
resonance equation (32). In short, I see no reason to change my opinion.

> Regards, Jim Peebles

This comment enhances Peebles' well-deserved reputation for astuteness, because there is no doubt that he has put his finger on the least-obvious and most questionable step in Parmenter's work, and I hope that Parmenter himself will elaborate in the next issue of *IE*; meanwhile, attempting to elucidate and justify this aspect of Parmenter's *magnum opus* provides me with an opportunity to bring to those interested in this matter what I regard as a truly excellent tutorial comment on some of the relevant background physics in a private e-mail debate which I had with my young friend Dr. Barry Merriman during March-July of 1997.

Dr. Merriman, an Assistant Prof. of Mathematics at UCLA, has for long been an accredited/recognized researcher at the DoE-funded [hot] Fusion Energy Research Program at UCSD. He is also open minded about the possibility that Low Energy Nuclear Transmutations (LENT), including Cold Fusion (CF), may be merely elusive rather than illusionary physical phenomena, and has given me several constructive suggestions [such as labeling a proffered talk to a physics department "lattice-catalyzed fusion" rather than CF in order to be less provocative]. He also acknowledges that a rigorous or highly plausible demonstration that LENT or CF is not truly forbidden would be an epochal scientific accomplishment, which [despite my status as an amateur who only knows some 1-D point-particle Quantum Mechanics (QM)] has spurred me to a considerable amount of work on the subject. In 1994 I gave Barry, in typescript,
a 1-page [Nat.Acad.Sci.-level], a 100-page [archival-journal, university-level], a 35-page [semi-popular, tutorial, college-level], and a 7-page [high-school-level] paper which, whatever their absolute merits, purport to: (a) be mathematically rigorous; (b) point out grievous flaws in the "best" published *CF impossibility 'proof'* known to me, namely that in the final half-dozen or so pages and exercises at the end of Chapter 1 of the admirable book on *QM* (Princeton Univ. Press) by the eminent Albert Einstein Prof. of Science at Princeton, Dr. P.J.E. Peebles; (c) patch-up genuine flaws in the pro-CF papers by the late Nobel Laureate in Physics, UCLA Prof. Julian Schwinger, which had been pointed out to me in a handwritten fax from another Nobel Laureate in Physics, Cambridge Univ. scientist Dr. Brian Josephson, around 1990 or 1991; (d) carry forward the line of thought initiated in 1989 in a Letter to *Physics Today* by LANL hot-fusion researcher Dr. Leaf Turner, which stimulated to deeper investigation CalPoly-Pomona Physics Prof., Dr. Bob Bush, from whom I heard about it at an ASME meeting in San Francisco in November 1989; (e) following the "be eclectic" urging of Fusion Information Center founder Dr. Hal Fox, incorporate or at least partly-overlap the better points in theories put forth through 1994, in alphabetic order, by retired BYU hot-fusion-researcher and Physics Prof., Bass [Ba]; by Bush [Bu]; the NRL uncle-nephew team of Drs. Talbott Chubb and Scott Chubb [C]; Purdue Nuclear Physicist Dr. Yeong Kim [K]; Univ. of Arizona Emeritus Prof. of Physics, Dr. Robert Parmenter [P] (a collaborator with erstwhile pro-CF colleague Dr. Willis Lamb, a Nobel Laureate in Physics); EPRI theoretical physicist Dr. Mario Rabinowitz [R];

Schwinger [S]; and Turner [T] (next to whom I had once sat for two weeks as a summer visitor to Los Alamos).

To dispose of the last item (e) first, recall that in 1994 I distributed widely a chart giving in the left-hand column 18 items which it seemed to me, any respectable CF theory should consider [although if I redid it now I would want to include investigations of lattice phonons by *CF Times* editor Dr. Mitchell Swartz and MIT EE Prof., Dr. Peter Hagelstein, etc.]. To save space I here list the items and, after each, the initials of those of the eight theorists of (e) who considered the item:

1. ZPF/LV (Zero Point Fluctuations & Lattice Vibrations): Ba, Bu, C, K, P, R, S, T.
2. Schwinger Ratio σ : Predicted Significance: S; Provided Λ for $\sigma = L/\Lambda$: C; First-Principles-Derivation Prediction: Ba.
3. Phonons: Fusion Heat Mediation and Inverse-Moessbauer-Effect Ion Excitation & De-Excitation: Ba, Bu, C, P, S.
4. QRT Ion Excitation: Resonant Non-Elastic-Collision Criterion, σ/π = ODD: Ba, Bu.
5. Globally Valid Potential $\boldsymbol{V(r)}$: OK Near Collision: Ba, C, K, P, R.
6. Velocity Distribution: Fusion Rate Enhancement: K, R; Resonance Line-Broadening: Ba, Bu.
7. Periodic $\boldsymbol{V(r)}$: In Solid-State Lattice: Ba, Bu, C, R, T.
8. Floquet-Bloch Theorem: $\{(\nabla\psi)/\psi\}$ <u>Required</u> Spatially Periodic: Ba, Bu, C, T.
9. Effective Δ-Mass: From Periodicity of $V(r)$: P, R.
10. Electron Screening: Fusion Rate Enhancement: Ba, C, K, P, R, S.
11. Madelung Forces: Fusion Rate Enhancement: Ba, C, P, S.
12. 3-D: 3-D Moessbauer-Analysis OK: C, P; Conduction Electrons in Host Lattice ($\Rightarrow \Lambda$): Ba, C, P.
13. Duane's Rule: For Inelastic Collisions & Resonant Transmission: Ba, Bu, C.
14. Resonant Transparency Energy Levels: Ba, Bu, P, T.
15. Nuclear Well Present: Ba, P.
16. Heat vs Loading Prediction: Bu.
17. Heat vs Current Prediction: Bush TRM Fine Structure: Ba, Bu.
18. QRT: Host-Lattice Suitability for Deuterons vs Protons Prediction: Ba.

The expert reader may wish at this point to read Appendix 1 and then inspect the MATLAB computer program in Figure 1 and look at the results of running the program eight times displayed in Table 1. Note that if one follows Parmenter and takes the width of a square-wave approximation to the Coulomb barrier to be the measured diameter of the influence of the strong nuclear force, the tunneling time is in femtoseconds; however, if one gradually takes the width to be 10 times, 100 times and then 1000 times larger, an amazing consequence of physical nonlinearity is revealed. When the width is taken to be 100 times larger, the tunneling time is in microseconds, but when an additional factor of 10 is used, then the tunneling time suddenly changes to scores or thousands of billions of years! This is an example of mathematical Catastrophe Theory (a tiny parameter change wreaks radical change in the predictions of the model) and/or Chaos Theory (incredible sensitivity to changes in initial and boundary conditions -- such as in the notorious Butterfly Effect in weather prediction or computed chaos in planetary dynamics).

Other readers may prefer to use the following debate between Bass and Merriman as a semi-popular tutorial which explains the derivation of the equations used in the MATLAB program.

What follows is an edited version of an e-exchange between Barry and myself from March 26, 1997 through the end of July, 1997. His words are prefaced by **M** and mine by **Ba**.

M: Resonant penetration simply takes a *long* time at low energies.

Ba: Did you not study my calculation of the penetration time when one includes in the calculation the ZPF and the uncertainty of the position of the bound deuterons?

M: Yes, I studied it in complete detail, and I fully understand your argument about ZPF line broadening. Further, that argument is probably correct, as far as it goes. The problem is your analysis is incomplete.

Ba: I came out that the tunneling time was only in picoseconds. Why do you continue to ignore this without refuting it?

MERRIMAN'S RESONANCE TUTORIAL

M: I neither ignore it nor refute it, because there is nothing wrong with that calculation. But we need to be clear on exactly what you have shown: what you show is that the uncertainty relationship dE dt > h implies that the time to complete the tunneling "must take at least" a few picoseconds, because dE is made bigger by ZPF broadening effects.

So, in effect, the uncertainty relationship is not a constraint on resonant penetration. I agree with you on that.

What you ignore is that the uncertainty relationship is *not the only time constraint* on the resonant tunneling process. The basic physics of resonant penetration imply a separate and much, much greater time constraint, totally independent of uncertainty issues.

Here is how resonant tunneling through a double barrier works, physically. I will consider in particular the case where the barriers are high and the waves are weak (since that is the case we are interested in). Each incoming wave is mostly reflected from the first barrier, but it transmits a small wave into the region between barriers. If the incoming waves are resonant with the space between the barriers, these small trapped internal waves add up constructively. In turn, these small internal waves are reflecting off the second barrier, transmitting a much, much smaller wave through that one. This second transmitted wave is observed as the net transmitted wave through the double barrier.

Once the internal trapped wave has resonantly built up to an *enormous* amplitude (much bigger than the amplitude of the incoming waves), it will be hitting the second barrier so hard that the wave transmitted through that barrier, while greatly reduced in amplitude compared to the internal waves, is as strong as the incoming waves. At this point, equilibrium is achieved and it looks like the incoming waves are perfectly transmitted through the double barrier.

So, the time it takes resonant penetration to occur is the time it takes the internal trapped wave to build up to a huge strength. Because it is built up from tiny contributions from each incoming wave, a huge number of incoming waves have to hit to reach this condition. *Thus, it takes a long time*. This is fundamental to the physics, and applies, for example, to systems such as water waves hitting a double obstruction like two parallel concrete "dams".

Based on this understanding, one can compute how long resonant penetration takes---assuming perfect resonance!---in any wave system, quantum or not, by figuring out how many incoming wave taps it takes to build up a huge internal wave strong enough to send a full amplitude transmitted wave through the second barrier.

Suppose a wave of amplitude A impinging on a single barrier makes a transmitted wave with amplitude fA, $f \ll 1$. In the double barrier case, these are internal waves and add constructively, so after N incoming waves have hit, the internal wave will have an amplitude A* = N fA. It in turn hits on the second barrier, which transmits a wave of amplitude f(A*) = f(NfA) = f^2 NA. In order for this to be the same amplitude as the incoming wave, we need A = f^2 N A, or N = 1/f^2 . This many waves must hit the double barrier for the (wave train)-(double barrier) system to reach the equilibrium where the net transmitted wave has the same amplitude as the incoming wave---*i.e.* the resonantly transparent state. Thus, the time it takes to achieve penetration is dt = N T, where T is the period of the waves.

In the quantum mechanical case, we have that the transmission factor f is the usual one-barrier tunneling factor, which goes like f ~ Exp(-w/w0), where w is the width of the barrier, and w0 is the wavelength of a particle with the same energy as the barrier height. Here, w is the width of the Coulomb barrier at the energy of the incoming particle (a few eV), which is roughly the Bohr radius, so, w ~ 10^-10 meters. Here w0 is the wavelength of a 10^6 eV particle, which is roughly about

300 x smaller, so f ~ Exp(-1000) ~ 10^-100, roughly. Thus, the number of wavelengths that must hit the barrier to achieve equilibration is N ~ 1/f^2 ~ 10^200. The period of these waves = the period of a ~ 1 eV particle, is T = 1/nu = h/h.nu = h/1eV ~ 10^-14 sec, so the time it takes to achieve resonant penetration is dt = N T ~ 10^200 * 10 ^-14 ~ 10^(several hundred) seconds. This is much longer than the current age of the universe, clearly.

What you must realize is that in the quantum case, there are two independent time constraints to consider: the first comes from achieving the resonant frequency, and that is controlled by the uncertainty principle. Your argument shows this time constraint is only picoseconds and thus of no consequence. But, even if the incoming waves have the *exact* resonant frequency, it also takes time to build up the internal trapped wave which is the mechanism by which transmission takes place. This latter process always takes a long time.

If you still cannot appreciate this, I will be glad to go over it in more detail---but you are a good physicist, so I think it must be pretty clear to you based on the above. This is not a subtle point.

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BASS's REPLY

Ba: As I told you previously, you have done me a big favor by working up a heuristic "proof" that my Resonant Transparency of the Coulomb Barrier theory, while theoretically correct, would take too long for the tunneling to be physically meaningful.

In your view, the tunneling would take longer than the age of the universe; this is the "Jaendel objection," independently made by Rabinowitz & Worledge, which I believe I have refuted thoroughly in the "long" version of my Line Broadening argument (you referred to it only in the short, Heisenberg Inequality version, which provides a mere *inequality* instead of an equality).

However, I accept the validity of your heuristic argument. I just don't believe that your rough assumptions about the numbers to enter into your formulae are the ones that are appropriate.

When I go over your argument with care, instead of the "eons" which you get, I still get picoseconds (as in my typed papers).

I shall review your formulae in sequence [Figure 1] & get [Table 1] picoseconds instead of eons (using essentially your own notation & argument above). The sole change in notation is that I shall quote your tunneling factor f from Stauffer & Stanley, *From Newton to Mandelbrot*, Springer, 1990, pages 93-94) as

f = exp[-2(a/L)],

where a is the width of the barrier and L is the de Broglie wavelength of the particle whose waves are beating on the barrier. The decision to use as a = R the measured diameter of an alpha particle, with a concomitant barrier height U_o = 447 keV, is taken in imitation of the beginning part of Parmenter's paper.

Barry, I have used your own formulae, but entered the appropriate constants much more carefully than you did. Do you agree now that I have demonstrated that *standard* quantum mechanics "predicts" Cold Fusion when one considers a free deuteron excited to an energy level of 17 eV inside of a lattice of bound deuterons?

Sincerely,

Bob

MERRIMAN's REJOINDER

M: We agree on the basic outline of the estimate. The only place we disagree is on the magnitude of "a", the effective width of the barrier. In my estimate, I said that "a" was on the order of a Bohr radius (~ 10^-10 meters), because that is the separation at which the Coulomb barrier starts to exceed the energy of the deuteron nuclease (~ 17 eV in your model), and thus that is the separation at which there is some sort of penetration barrier. Instead, you say above that "a" is about 100,000 times smaller, basically the separation scale at which the barrier assumes its peak height of ~ 20 MeV. The truth is obviously somewhere in between---the effective barrier is wider than the sort of "width at half maximum" you use above, and narrower than the "atomic separation width" I use.

My claim is that the truth is closer to my "a" than yours. You have computed the realistic Madelung potential, so in principal you can decide what its effective width is based on that. You say it is steeper than the basic Coulomb barrier---but I can't believe it's thousands of times steeper. If it is, I would agree with your estimate above, since the barrier would be thousands of times thinner than my estimate. But I bet even your detailed potentials have a barrier width of ~ 0.01 Angstrom if we measure the width where its height is 10,000 eV. If you do the tunneling time estimate above based on how long it would take the 17 eV particle to tunnel through a 10,000 eV barrier of thickness a = the width of your Madelung potential barrier at the 10,000 eV level, this would provide an underestimate of the penetration time (since the full barrier is at least as high and thick). I bet this would yield a very long time already, much much longer than the femtosecond times above.

More generally, if we let dt(a,U0) denote the estimated time to resonantly tunnel through a pair of barriers of height U0 and width a, as outlined above, what you showed is that dt(10^-15 m, 20 MeV) ~ femtoseconds, while I then showed that dt(10^-10 m, 20 MeV) ~ many many years. I think that if we let a(U) = width of the Coulomb barrier at height U, then dt(a(U),U-E) is always an underestimate of the time it takes the particle of energy E to penetrate (since the true barrier is at least as wide and high) & so the max over these estimates over all 0 < U < 20 MeV would give a decent simple estimate for the true tunneling time, and any one value sets a new lower bound. Of course, the real penetration factor f is some integral over the barrier which can be carried out---not sure if you have done so. The procedure just mentioned gives a simpler and cruder way to get a reasonable estimate.

In any case, we should argue about the appropriate value of "a" until we come to some agreement. Also, as you mentioned, you are free to divide the penetration time by the number of deuterons in the whole system, since even single fusion events would be interesting and detectable----thus you have a factor of ~ 10^16---10^23 to play with, but I suspect even that will not be enough to bring it down to a reasonable time.

RELEVANCE TO PARMENTER'S PAPER

Ba: I hope that the reader will find the preceding "unfinished debate" enlightening as a partial background to the Parmenter paper.

To my mind, the greatest achievement of Parmenter is that he has considered the problem of two nearby deuterons in their full three-dimensional and QM & QED & Nuclear-Physics detail. He shows convincingly that the nuclear reaction channels observed in fusion-temperature plasmas and in (gaseous) "Cold Fusion" of the Muon-Catalyzed type [studied by BYU Prof. of Physics, Dr. Steve Jones] are for the beyond-beta-phase ultra-loading scenario he (like Schwinger) has chosen to analyze, enormously less likely than the supposedly "impossible" d + d => He^4 observed by the most dedicated CF experimentalists. Also, Parmenter's theory is truly "scientific" in the sense of Popper, because, like all really valuable scientific theories, his theory makes highly specific predictions (observable by nuclear magnetic resonance testing of a working CF cell) which can be *"falsified if false."* Therefore if Parmenter's theory is fundamentally wrong, it would be quite possible to establish its failure by simply doing some measurements that have not been made to date!

It seems noteworthy that a genuine expert like Peebles finds no fault with the first 24 equations of Parmenter (which I regard as perhaps the paper's most enduring contribution), and endorses the linch-pin equation (25), but balks only at the numerical values in equation (27). There are two possible reasons which I can suggest as explanations. Peebles may have read the earlier papers by Parmenter & Lamb, and not agreed with some point; but I suspect it is more likely that, since those papers are in the *Proceedings of the National Academy of Science* to whose pages a Member like Lamb has automatic access without the hindrance of anonymous referees, Peebles may suspect that this work has not actually passed conventional "peer review" and therefore may be questioned as much as any self-published "Preprints" by unestablished wannabes like myself. However, this is not a fatal quibble, because the thrust of the earlier P&L papers was to arrive at a number consistent with published data; accordingly, it must be possible to justify the numbers in (27) by short *ad hoc* references to experimental data. Prof. Peebles' relentless skepticism is perfectly good science [Sir Arthur C. Clarke quotes Carl Sagan's dictum that *extraordinary claims require extraordinary proof*], and I must confess that I too was puzzled when I first came upon (32). It is evident to me (who would like to be convinced!) that Parmenter must have some background knowledge pertaining to resonance that is one of the *lacunae* in my own education, because he justifies (32) by (33) and I have not the faintest clue why (33) is relevant to the validity of (32)! Hopefully in the next issue of *IE* this matter will be elucidated by Dr. Parmenter.

Nevertheless I have satisfied myself that (32) is physically plausible, by the following reasoning. Consider the well-known closed-form solution to a resonantly-forced harmonic oscillator

$$d^2\psi/dt^2 + \omega_o^2\psi = -\gamma \sin(\omega_o t),$$

and discard the [asymptotically irrelevant] bounded solutions in favor of the unique unbounded solution, whose maximum (after N bangs, when examined stroboscopically, at times $T_N = (2\pi/\omega_o)N$), is $(\gamma/[2\omega_o])(2\pi/\omega_o)N$, i.e. the maximum

$$\psi_N = (\gamma/[2\omega_o])\, T_N = N\psi_1, \qquad \psi_1 = \gamma(\pi/\omega_o^2),$$

grows linearly with time, and in fact *increases* by a *fixed amount*, ψ_1, after each bang. But this is *precisely* what (32) claims, when one notes that now

$$\{\omega_o T/[2\pi]\} = N.$$

Moreover, it is *reasonable* to take the two deuterons as behaving like a harmonic oscillator, because their far-potential is dominated by the term quadratic in r when the deuterons are trapped inside the tetrahedral cavity specified by Parmenter, and so the wavefunction ψ "increases by a small amount each time the two deuterons bang into the [near-potential] Coulomb barrier separating them (with the characteristic angular frequency $\omega_o = 4.456 \times 10^{14}$ per sec)."

R [cm]	dt [sec]
2.9975×10^{-13}	3.2027×10^{-15} = 3.2 femtosec
3.2200×10^{-13}	3.5181×10^{-15} = 3.5 femtosec
2.9975×10^{-12}	8.4208×10^{-13} = 0.8 picosec
3.2200×10^{-12}	1.1332×10^{-12} = 1.1 picosec
2.9975×10^{-11}	3.6331×10^{-5} = 36 microsec
3.2200×10^{-11}	9.2480×10^{-5} = 93 microsec
2.9975×10^{-10}	1.4202×10^{19} = 45 billion years
3.2200×10^{-10}	2.3467×10^{20} = 7,441 billion years

TABLE 1


```
function  [dt] = res_tunnel_01(R,E);
c = 3e008;                                    %  speed of light in m/sec
h = 4.136e-0015;                              %  Planck's constant in eV-sec
Md_bycsq = 2*938.3*1e006;                     %  deuteron mass times c^2 in eV
Md = Md_bycsq/(c^2);                          %  deuteron mass in eV
el =  R;                              %  input potential barrier width in m
Ro = 3.22e-0015;                              %  diameter of alpha particle in m
Uo = 447000*(Ro/el);                          %  height of potential barrier
Eo = E;                                       %  input energy of excited deuteron in eV
lambda = h/(2*pi*sqrt(2*Md*(Uo - Eo)));       %  de Broglie wavelength of excited deuteron
f = exp(-2*(el/lambda));                      % quantum-mechanical barrier penetration factor
N = 1/f^2;                                    % number of bangs to penetrate
T = h/Eo;                                     %  period of bangs against barrier
dt = N*T;                                     %  time to penetrate Coulomb barrier in sec
```

FIGURE 1

Appendix 1

RESONANT TRANSPARENCY SPECTRUM OF COULOMB BARRIERS

In a poster paper at ICCF-5, I presented a more rigorous version of the following simplified quantum-mechanical derivation of the Spectrum of Energy Levels for a low-energy excited deuteron [energy between 6 eV and 150 eV] to find the Coulomb barriers of the two closest adjacent lattice-bound deuterons resonantly transparent.

It is known that a linear model of a 3-D lattice is adequate to predict the Moessbauer Effect, in which the recoil of a single nucleus affects an entire crystal. Hence I have simplified the metallic lattice in which hydrogenic nuclides (protons or deuterons) are embedded from a 3-D model to a linear model. I have in mind a lattice of either palladium ions or nickel ions, but these do not appear explicitly; they serve as a ghostly background which establishes rigidly the distance between the positively-charged ions of interest. Therefore the only lattice analyzed is a proton or deuteron lattice.

First, consider a straight line $-\infty < r < +\infty$, with an excited particle located near $r = 0$. Next, place a bound positive unit charge at $r = -L$ and another at $r = +L$. The excited particle then experiences a repulsive potential

$V(r) = e^2\{ [1/|r + L|] + [1/|L - r|] \} = 2e^2\{ L/(L^2 - r^2) \}$,

which resembles an infinitely high "spike" as the particle moves left toward $r = -L$ or right toward $r = +L$. Now imagine this potential repeated periodically so that there are spikes at $r = j.L$, for every positive or negative integer $j = \pm 1, \pm 2, \pm 3, \ldots$. The objective is to find conditions under which the excited particle finds all of these barriers "resonantly transparent," so that it can travel from left to right as if there were no barriers at all.

For present purposes, I shall simplify even this simplified model, and replace each of the spikes by a rectangular potential of finite height U_o and width $l << L$. Furthermore I shall consider only the central potential well and its immediate vicinity to left and right; therefore, we can shift the origin to the first barrier, and take the potential to be $V = 0$ in *region I*, $r < 0$, while it will be $V = U_o$ in *region II*, $0 < r < l$, and again $V = 0$ in *region III*, the potential well of interest, defined to be $l < r < L + l$. Similarly, the second barrier is defined as $V = U_o$ in *region IV*,

$L + l < r < L + 2l$. Finally, we define *region V* as $L + 2l < r$, wherein once again $V = 0$.

I shall now derive quite rigorously from Schroedinger's wave mechanics and this crude model a physical result which is identical to what I presented at ICCF-5 using a far more realistic potential. In that work, I place bound positive charges at every location $r = j.L$, for $j = \pm1, \pm2, \pm3, \ldots$ and then model the effect of the circulating electrons (and *average* electrical neutrality) by placing bound negative charges half-way between every such pair of bound positive charges, except on the interval $-L < r < +L$. When one writes down the infinite series of these potentials, it turns out that they can be summed in closed form (analogous to the example above). It is physically required to consider all of these charges, because the ions are rigidly bound, and the electrons (by symmetry and neutrality considerations) *on average* may be treated as if bound (except in the central interval), and because electrostatic forces are long-range forces which in principle cannot really be screened out and must not be ignored [when their positions are fixed]. In this way I got a potential $V = V(r)$ which is a Coulomb/Madelung potential. However, the line is not yet electrically neutral, because 3 electrons are missing, namely those on the central interval near $r = -L$, $r = 0$, and $r = +L$. From the papers of Parmenter & Lamb (and their references to the book of Mott), I learned how to include these final 3 charges as a "smeared out electron cloud" of negative charge distributed uniformly over the central interval, which results in a harmonic restoring force or quadratic potential $W = \omega_o^2(r^2/2)$ proportional to r^2, where the constant of proportionality, ω_o^2, is so chosen as to represent the effect of 3 electron charges. The final result is a Coulomb/Madelung/Fermi-Thomas/Mott potential $V(r) = V_{C/M/FT/M}(r)$ which is defined only on the central interval $-L < r < +L$, but is then extended by definition to apply as a *periodic* potential $V(r) \equiv V(r + j.L)$, $j = \pm1, \pm2, \pm3, \ldots$. At this point I use what Drs. Scott Chubb & Talbott Chubb of the NRL refer to as the central result of solid-state physics, *Bloch's Theorem*, according to which no solution of Schroedinger's Equation is relevant in the present context unless its logarithmic derivative is spatially periodic of the same period as the potential. I am satisfied that the potential $V_{C/M/FT/M}$ derived as just explained is extremely accurate, because I have tested it as follows.

It is well-known in quantum physics that even at absolute zero temperature the bound ions in a metallic lattice continue to fluctuate in position, with what are called Zero Point Energy (ZPE) motions, or as a manifestation of Zero Point Fluctuations (ZPF). The rms amplitude Λ of such ZPF vibrations can be measured experimentally (e.g. by blurs on an x-ray, or from neutron diffraction studies). I call the dimensionless ratio

$\sigma = (L/\Lambda)$

the Schwinger Ratio, in honor of the late Nobel Laureate physicist (and Cold Fusion proponent) Dr. Julian Schwinger, because in his published and unpublished papers on CF he in effect conjectured that this ratio was all-important [and the possible significance of Λ for CF had also been pointed out in early CF studies by Dr. Mario Rabinowitz]. I have tested the validity of my periodic $V_{C/M/FT/M}$ potential by deriving from it a prediction of the value of the empirical Schwinger Ratio (which is about 22 for a beta-phase palladium lattice embedded with deuterons), and then confirming that the prediction is within one-third of one percent of measured reality!

Note that the Schwinger ratio includes the effect of the metallic *host lattice* (which establishes L) and also the effect of the mass M of the particle (whether it is a proton [M/2] or deuteron [M] affects Λ).

Therefore there is a definitively conclusive way to test the validity of my potential, which I call the *Rabinowitz acid test* because my good friend Mario had mistakenly opined that no known CF theory could pass this test. My *Quantum Resonance Triggering* (QRT) Criterion for CF with a given *host-lattice/particle* pair, whose foundation will be derived below, is that the pair is suitable for CF if and only *if σ/π is closer to an ODD than an even integer*. If one considers the host lattice to be either palladium or nickel, and the embedded particles to be either protons or deuterons, then there are 4 distinct possibilities. My QRT Criterion predicts that palladium will work well only with deuterons (thus justifying the use of protons as a "control" in Fleischmann-Pons types of electrolytic cells), while, conversely, protons will work better with nickel! This is because σ/π is a highly nonlinear function of the particle mass, whose value jumps from odd to even when in the case of a palladium host-lattice, the mass M of the particle [a deuteron] is divided by two to give the approximate mass of a proton!

The resonant transparency criterion to be derived below leads to one of my two derivations of the QRT Criterion, and is closely related to the computation of the energy levels of bound particles in a lattice. Using the numerical methodology given in a book of Dr. Steven Koonin for the latter, I have with $V_{C/M/FT/M}$ computed numerically the lowest 600 energy levels

$E_n = f(n, \sigma)$, $(n = 0, 1, 2, 3, \dots , 600)$,

[which ranges from about 6 eV to about 150 eV] of the Spectrum of Resonant Transparency for deuterons embedded in a palladium host lattice. I have also validated Schwinger's Conjecture by proving rigorously that this spectrum is a function of *nothing* but the fundamental constants of physics and the Schwinger Ratio σ.

The following Theorem will be proved here with the simplified potential defined above; however, it has also been proved (using the WBK approximate/asymptotic solution of Schroedinger's equation, which becomes numerically indistinguishable from the exact solution for all values of n other than the smallest few) in the case of the realistic periodic $V_{C/M/FT/M}$ potential described above. (I no longer have copies of my CF papers, but I once gave Dr. Barry Merriman of UCLA a complete set for safe-keeping.)

THEOREM. Consider two rectangular [square-wave] potential barriers, each of arbitrarily great fixed height U_o and arbitrarily small width l, and spaced an arbitrary distance L apart. Suppose that a subatomic particle of mass M and kinetic energy $E < U_o$ impinges on the left-hand barrier while moving uniformly toward the right. Then [disregarding an asymptotically unimportant small bias which is relevant only for the smallest few 'multiples' mentioned below] a *necessary and sufficient condition* for the particle to tunnel quantum-mechanically through both barriers and emerge from the right-hand barrier still moving toward the right with the same energy is that the "well-width" distance L between the barriers should be an *odd integral multiple* of the *particle's quarter de Broglie wavelength* ($\lambda/4$), where $\lambda = h/\{2\pi(2M.E)^{1/2}\}$ and h is Planck's constant.

PROOF. As explained in introductory physics texts, the probability density of a particle is given by $|\psi|^2$, where the complex amplitude ψ is a solution of Helmholtz's Equation (the same equation which describes the configuration of standing sound-waves in pipe organs and which predicts the nature of standing electromagnetic waves in cavity resonators such as microwave ovens). After factoring out the time-behavior in the form of a sinusoidally-oscillating factor $\exp(2\pi \boldsymbol{i}\nu t)$, where the frequency $\nu = E/h = 1/\tau$, the spatial behavior is governed by

$\psi'' + k^2\psi = 0, \quad k^2 = (8\pi^2 M/h^2)[E - U]$,

so that $\psi = \exp(\pm \boldsymbol{i}kr)$, depending upon whether the positive or negative square-root is chosen for the wave-number k. If we let j = 1, 3, 5 in *regions I, III, V* and let j = 2, 4 in *regions II, IV* as defined above, then there will be 10 constants (a_j ,b_j) such that

$\psi_j = a_j \exp(\boldsymbol{i}kr) + b_j \exp(-\boldsymbol{i}kr)$, (j = 1, 3, 5),

$\psi_j = a_j \exp(\kappa r) + b_j \exp(-\kappa r)$, (j = 2,4),

$k = (2\pi/h)(2M.E)^{1/2} = 1/\lambda$, $\kappa = (2\pi/h)(2M.[U - E])^{1/2}$,

and where, in the odd-numbered regions, the terms associated with the b_j coefficients provide (in the light of the time-factors) the waves which are traveling toward the right. Now define column 2-vectors $c^j = [a_j , b_j]' =$

$[a_j ; b_j]$, where (as in MATLAB notation), apostrophes denote vector-matrix transposition of rows and columns, and where the semi-colon denotes stacking the next item in the next row (beneath the present row). As in pages 721-724 of Joos, *Theoretical Physics*, Dover (1986), the function ψ and its first derivative $d\psi/dr \equiv$

ψ' must be continuous at the 4 boundaries between the 5 regions, resulting in algebraic equations of the form
$M_j c^j = M_{j+1} c^{j+1}$, (j = 1, 2, 3, 4),
$M_1 = [1, 1; \boldsymbol{i}k, -\boldsymbol{i}k]$, $M_2 = [1, 1; \kappa, -\kappa]$,
$M_3 = [\exp(\kappa l), \exp(-\kappa l); \kappa.\exp(\kappa l), -\kappa.\exp(-\kappa l)]$,
$M_4 = [\exp(\boldsymbol{i}kl), \exp(-\boldsymbol{i}kl); \boldsymbol{i}k.\exp(\boldsymbol{i}kl), -\boldsymbol{i}k.\exp(-\boldsymbol{i}kl)]$,
$M_5 = [\exp(\boldsymbol{i}\mathbf{k}.[L+ l]), \exp(-\boldsymbol{i}k.[L+l]); \boldsymbol{i}k.\exp(\boldsymbol{i}\mathbf{k}.[L+ l]), -\boldsymbol{i}k.\exp(-\boldsymbol{i}\mathbf{k}.[L+ l])]$,
$M_6 = [\exp(\kappa.[L+ l]), \exp(-\kappa.[L+ l]); \kappa.\exp(\kappa.[L+ l]), -\kappa.\exp(-\kappa[L+ l])]$,
$M_7 = [\exp(\kappa[L+2l]), \exp(-\kappa[L + 2l]); \kappa.\exp(\kappa[L+2l]), -\kappa.\exp(-\kappa[L + 2l])]$,
$M_8 = [0, \exp(-\boldsymbol{i}k[L + 2l]); 0, -\boldsymbol{i}k.\exp(-\boldsymbol{i}k[L + 2l])]$,
$c^1 = Mc^5$, $\quad M = \Pi_1^{\;7}\{M_j^{-1} M_{j+1}\}$,
where in the repeated matrix-product only odd values of j are allowed. The wave in the *region V* is to be purely a transmitted, right-traveling wave, so that $c^5 = [0, 1]'$. Hence c^1 is well-defined and determined uniquely. After scores of pages of tedious algebra, one finds a closed-form solution for b_1 which must be set equal to unity if complete resonant transmission through both barriers is to be achieved. Separating the real and imaginary components of this complex equation, one obtains, finally
$\tan(k.L) = -\tan(\beta)$,
where β is a function only of (k, κ, l) but not L. Accordingly,
$k.L = \beta + (4n + 1)(\pi/2)$, whence, finally, recalling the definition of k,
$L = L_o + (4n + 1)(\lambda/4)$
as claimed.

COROLLARY. If the well-width L is fixed, then the Spectrum of Energy Levels corresponding to resonant transparency is given by
$E_n = E_o.(4n + 1)^2$, (n = 1, 2, 3, …),
where $E_o = \{[h/(L - L_o)]^2\}/(32M)$.